

Build Selector Table

Review: LL(1) SDT

Build the selector table for this grammar

Grammar

- ① $S ::= \text{lpar } X \text{ rpar}$
- ② $X ::= \text{id comma } X$
- ③ $X ::= \varepsilon$

$\text{FIRST}(S) = \{ \text{lpar} \}$

$\text{FIRST}(X) = \{ \varepsilon, \text{id} \}$

$\text{FOLLOW}(S) = \{ \text{eof} \}$

$\text{FOLLOW}(X) = \{ \text{rpar} \}$

Building Selector Tables

for each production $X ::= \alpha$

for each t in $\text{FIRST}(\alpha)$

put $X ::= \alpha$ in $\text{Table}[X][t]$

if ε is in $\text{FIRST}(\alpha)$

for each t in $\text{FOLLOW}(X)$

put $X ::= \alpha$ in $\text{Table}[X][t]$

	lpar	id	comma	rpar	eof
S					
X					

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S					
X					

Administrivia

Housekeeping

Review

Catch-Up

Actions Preserved Through Transforms

SDT for Top-Down Parsing

Augmented CFG

$E ::= E \text{ minus } T \#1$
 $\quad \quad \quad | T$
 $T ::= \#2 \text{ intlit}$

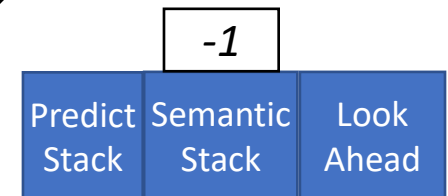
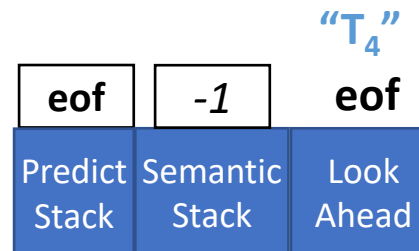
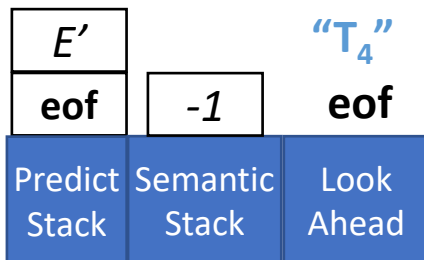
	intlit	minus	EOF
E	$T E'$		
E'		$\text{minus } T \#1 E'$	ϵ
T	$\#2 \text{ intlit}$		

Eval Stack Actions

$\#1$ $tTrans = \text{sem.pop}()$;
 $\quad eTrans = \text{sem.pop}()$;
 $\quad LTrans = eTrans - tTrans$;
 $\quad \text{sem.push}(LTrans)$
 $\#2$ $\text{sem.push}(\text{look.value})$

Token Stream: intlit:7 minus intlit:8 eof
 T_1 T_2 T_3 T_4

Accept!



Neat Trick! So What?

SDT for Top-Down Parsing

Expression evaluation is great and all...

- But what we really want is a translation to an AST
- Let's see a similar example



What About Producing ASTs?

SDT for Top-Down Parsing

Augmented CFG

$E ::= E + T \#1$
 $| T$
 $T ::= \#2 \text{ intlit}$

	intlit	+	EOF
E	$T E'$		
E'		$+ T \#1 E'$	ϵ
T	$\#2 \text{ intlit}$		

Take this out

Eval Stack Actions

```
#1 tTrans = sem.pop() ;  
   eTrans = sem.pop() ;  
   LTrans = eTrans + tTrans;  
   sem.push(LTrans)  
#2 sem.push(intlit.value)
```

Put this in

AST Building Stack Actions

```
#1 tTrans = sem.pop() ;  
   eTrans = sem.pop() ;  
   LTrans = PlusNode(eTrans,tTrans)  
   sem.push(LTrans)  
#2 LTrans = IntLitNode(intlit.value)  
   sem.push(LTrans)
```

What About the AST?

SDT for Top-Down Parsing

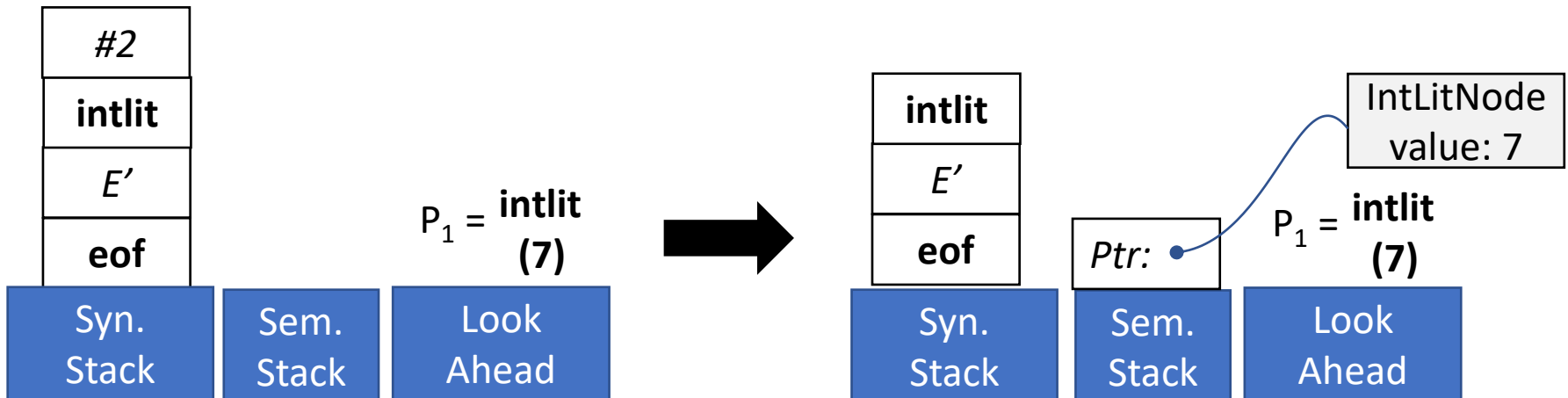
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E	$T E'$		
E'		$+ T \#1 E'$	ϵ
T	$\#2 \text{intlit}$		

AST Building Stack Actions

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   sem.push(LTrans)  
  
#2 LTrans = IntLitNode(intlit.value)  
   sem.push(LTrans)
```

Token Stream: 7 + 8 + 9





A FEW

Transitions

LATER

What About the AST?

SDT for Top-Down Parsing

Selector Table

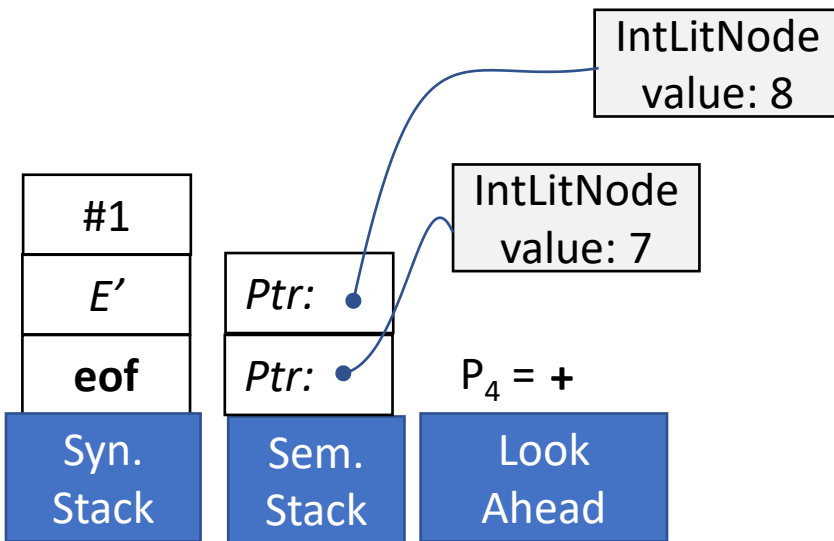
	intlrit	+	EOF
E	$T E'$		
E'		$+ T \#1 E'$	ϵ
T	$\#2 \text{intlrit}$		

AST Building Stack Actions

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   sem.push(LTrans)
    
```

Input: 7 + 8 + 9



What About the AST?

SDT for Top-Down Parsing

Selector Table

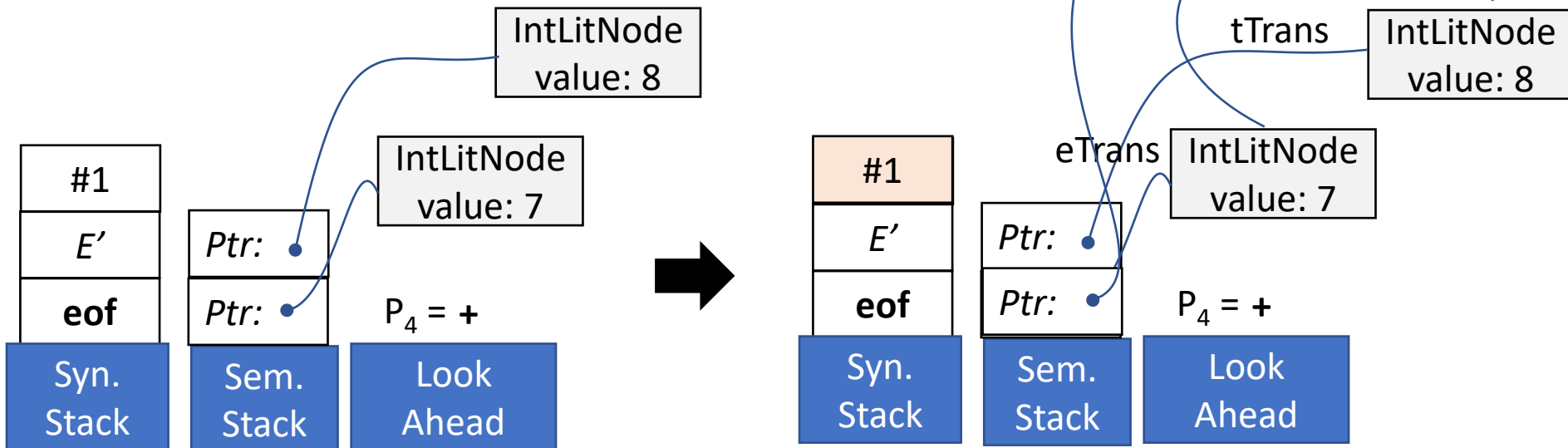
	intlrit	+	EOF
<i>E</i>	<i>T E'</i>		
<i>E'</i>		<i>+ T #1 E'</i>	ϵ
<i>T</i>	<i>#2 intlrit</i>		

AST Building Stack Actions

```
#1 tTrans = sem.pop() ;
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   LTrans = PlusNode(eTrans,tTrans)
   sem.push(LTrans)

#2 LTrans = IntLitNode(intlrit.value)
   sem.push(LTrans)
```

Input: 7 + 8 + 9



This is us “Fixing” the Tree

SDT for Top-Down Parsing

“We’ll fix the tree later”

— _ (ツ) _ / —

The parse tree is busted, but the AST comes out A-ok!

Summary

SDT for Top-Down Parsing

We've Completed one (1) way of deriving a parse tree:

- Added action numbers to grammar
- Converted SDD rules to stack actions
- Built an LL(1) parser from the grammar

Next Time

SDT for Top-Down Parsing

Handling more complicated languages

- Moving beyond the limitations of LL(1)



EECS 665

COMPIILER

CONSTRUCTION

LR Parsing

Last Time

Review LL(1) SDT

LL(1) SDT

- Add 2nd stack for symbol translations
- Turn SDD to semantic stack actions
- Embed triggers into syntactic stack

You should know

How to convert from SDD to stack actions

How to run SDT for an LL(1) parser



Parsing

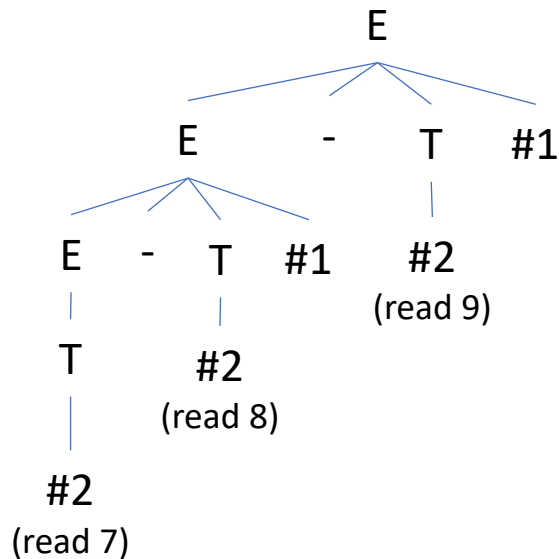
Actions Preserved Through Transforms

3 Grammars for Subtraction Expression:

intlit(7) – intlit(8) – intlit(9)

Augmented Left-Assoc CFG

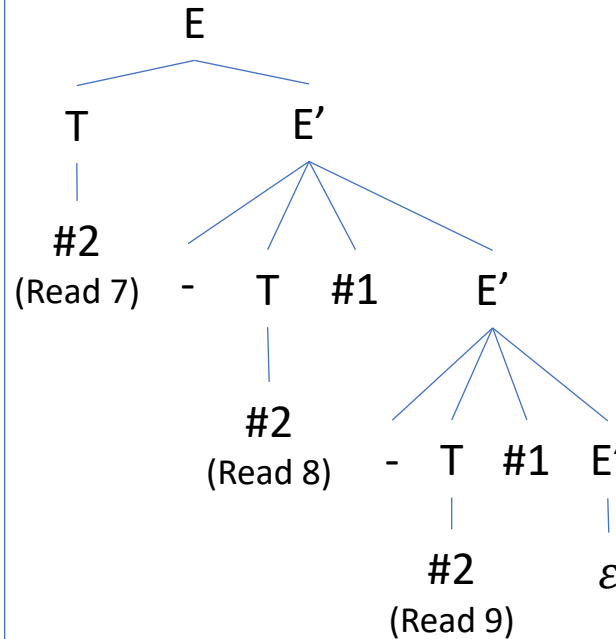
$E ::= E - T \#1$
| T
 $T ::= \#2 \text{ intlit}$



#2 #2 #1 #2 #1
7 8 -1 9 -10

LL(1) CFG

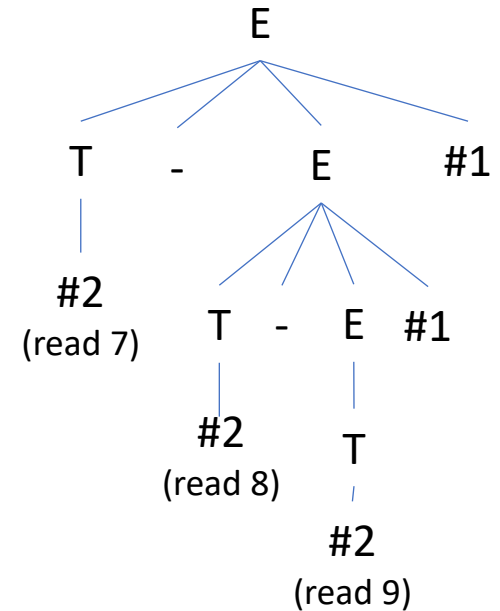
$E ::= T E'$
 $E' ::= - T \#1 E' \mid \epsilon$
 $T ::= \#2 \text{ intlit}$



#2 #2 #1 #2 #1
7 8 -1 9 -10

Augmented Right-Assoc CFG

$E ::= T - E \#1$
| T
 $T ::= \#2 \text{ intlit}$



#2 #2 #2 #1 #1
7 8 9 -1 -10

8

Today's Outline

Lecture 11 – LR Parsing

LL(1) Wrap-up

- Limitations

LR Parsers

- Concept
- Theory
- Operation



Parsing

LL(1) Summary

LL(1) Wrap-Up

Predictive Parser

- Commits to an entire production based on observed lookahead. For LL(1), predict a production based on 1 token of lookahead

Implementation

- Stack and a single streamable token

Many Useful Languages are NOT LL(1)

LL(1) Wrap-Up

Could increase lookahead

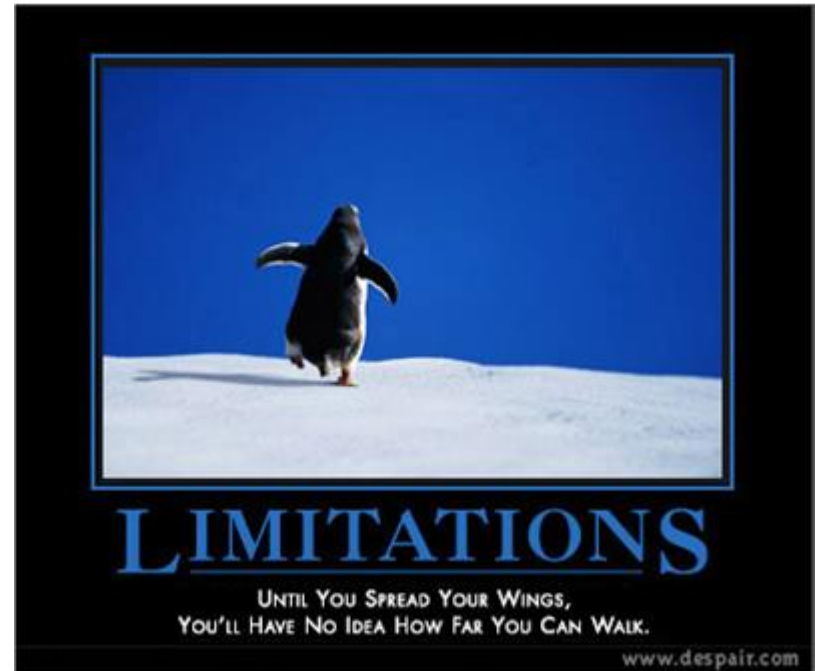
- Up until the mid 90s, this was considered impractical

Could add runtime complexity

- CYK has us covered there

Could add memory complexity

- i.e. more elaborate parse table



LR Parsers

LR Parsers - Concept

Advantages

- Suitable for most programming languages
- Runtime: time and space $O(n)$ in the input size
- Subsumes corresponding LL parser i.e. $LL(1) < LR(1)$
- $LR(1)$ parsers can recognize any Deterministic CFG

Disadvantages

- More complex parser generation
- Larger memory state

Linear Parsers: Commitment Issues

LR Parsers - Concept

Linear Parser have to commit to their parse tree in a linear scan

LL(1) Parser:
commits ASAP



Backtracking Parsers:
Go back on commitments



LR Parsers:
Delay commitments



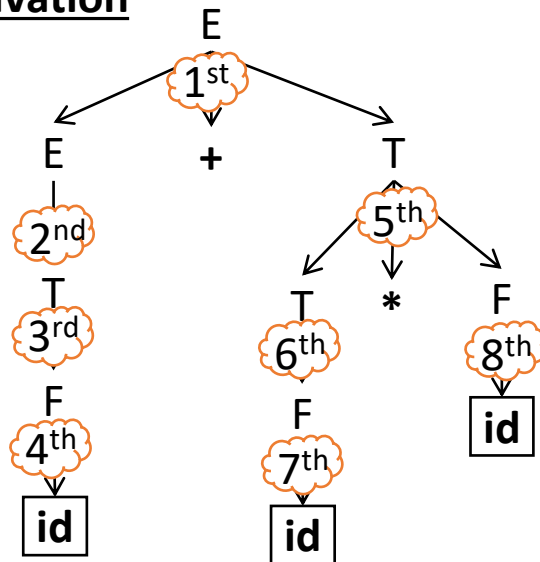
Delayed Commitment to a Rule

LR Parsers - Concept

Can we choose a production *after* we've seen it's subtree?

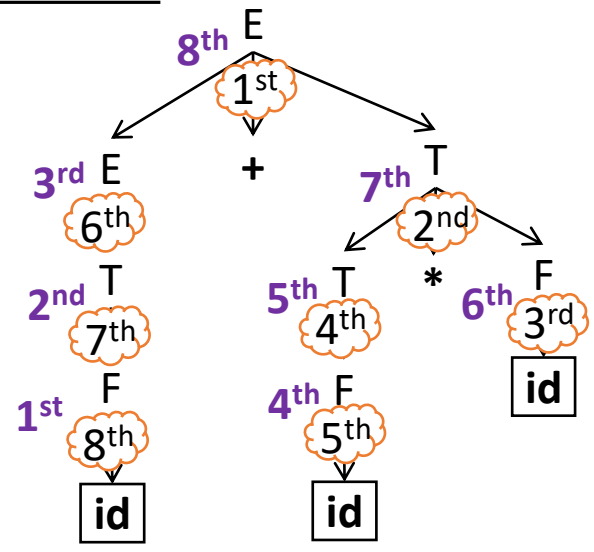
- Yes, if we build the tree *bottom-up*!
- What about derivation order?

Leftmost Derivation



Reverse

Rightmost Derivation



LR(k) Parsers

LR Parsers - Concept

Perform reverse rightmost derivation

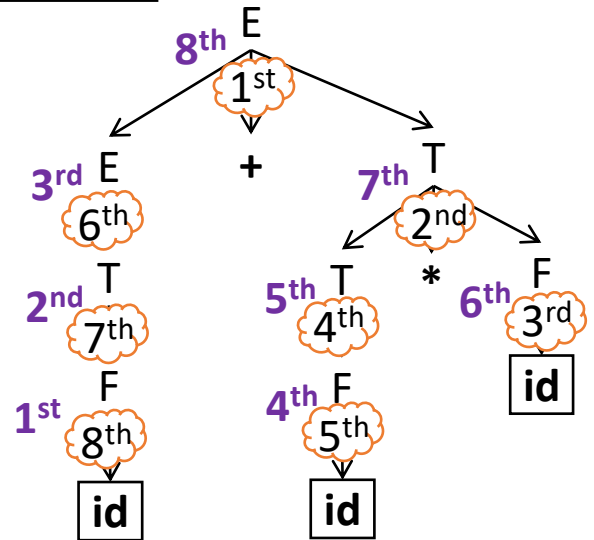
- Always move up branch before starting new branch
- Work on the LAST SYMBOL(S) of the derivation PREFIX

Reverse Rightmost derivation

$$\begin{aligned} E &\Rightarrow E + T \\ &\Rightarrow E + T * F \\ &\Rightarrow E + T * \text{id} \\ &\Rightarrow E + F * \text{id} \\ &\Rightarrow E + \text{id} * \text{id} \\ &\Rightarrow T + \text{id} * \text{id} \\ &\Rightarrow F + \text{id} * \text{id} \\ &\Rightarrow \text{id} + \text{id} * \text{id} \end{aligned}$$

Reverse

Rightmost Derivation



LR(k) Parsers

LR Parsers - Concept

Perform reverse rightmost derivation

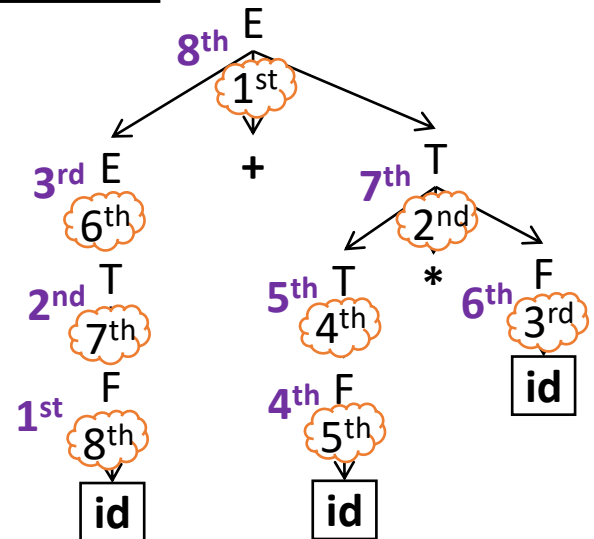
- Always move up branch before starting new branch
- Work on the LAST SYMBOL(S) of the derivation PREFIX

Reverse Rightmost derivation

id| + id * id
F| + id * id
T| + id * id
E + id| * id
E + F| * id
E + T * id|
E + T * F|
E + T| ← E

Reverse

Rightmost Derivation



LL vs LR Parsers

Beyond LL(1)

LL Operations

- *Predict* the rule used to expand nonterminal
- *Verify* that a terminal was expected

LR Operations

- *Shift* to a new branch (add a new leaf)
- *Reduce* a string of symbols to the parent nonterminal

Bottom-Up Parsing – Example

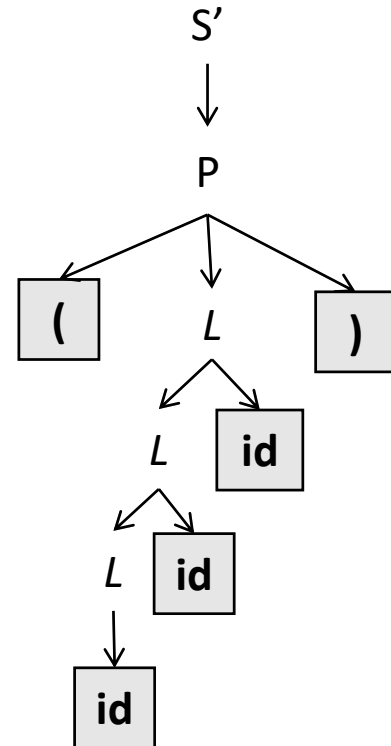
LR Parser Construction

<u>Prefix</u>	<u>Suffix</u>	<u>Action</u>
	(id id id) eof	shift (
(	id id id) eof	shift id
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(L	id id) eof	shift id
(L id	id) eof	Red. ④ $L ::= L id$
(L	id) eof	shift id
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(L	) eof	shift)
(L)	eof	
P	eof	Red. ② $L ::= (L)$
S'	eof	Red. ① $S' \rightarrow P$

Accept!!

- Grammar G
- ① $S' ::= P$
 - ② $P ::= (L)$
 - ③ $L ::= id$
 - ④ $L ::= L id$

Correct Parse Tree

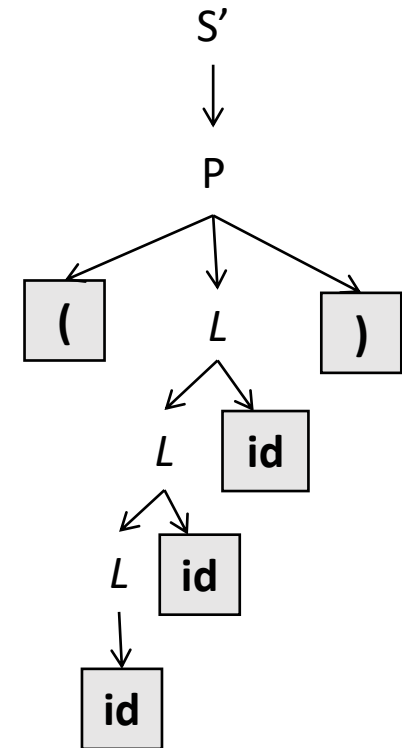


LR Parser – Towards Implementation

LR Parser Construction

Key Points

- Relies on *shift* and *reduce* actions
 - Shift – add a **terminal** leaf to parse tree
 - Reduce – add nonterminal to parse tree
- Builds the parse tree *bottom-up*
 - *Does as many reduce actions as possible before a shift*

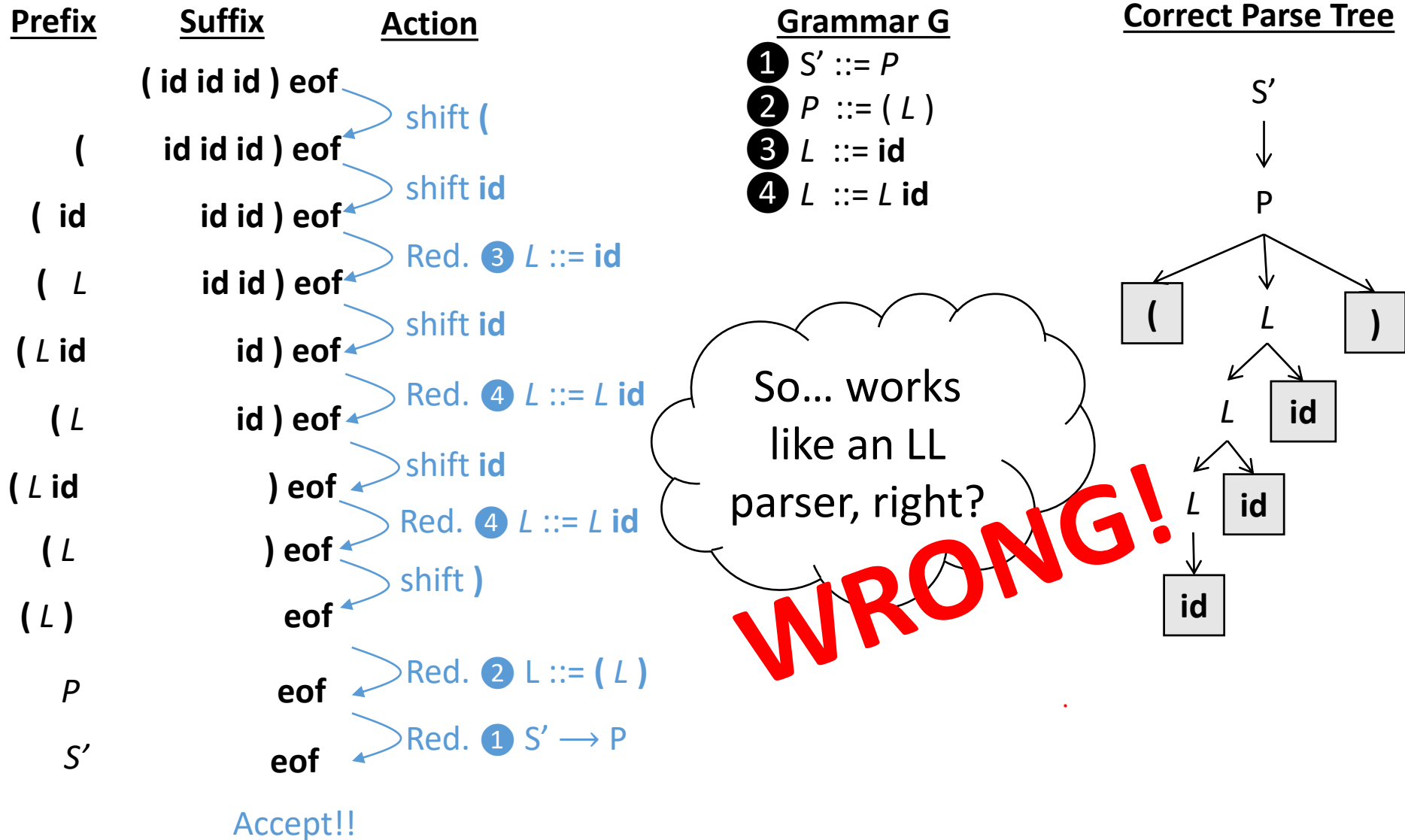


Grammar G

- 1 $S' ::= P$
- 2 $P ::= (L)$
- 3 $L ::= id$
- 4 $L ::= L id$

Bottom-Up Parsing – Implementation

LR Parser Construction



LL vs LR Parser – We are Not the Same

LR Parser Construction

LL(1) - Uses 1 token of lookahead and 1 nonterminal to pick production

“Oh, I’ll just use that same info for the LR(1) parser”



LR Parser – Towards Implementation

My darkest timeline fanatic:

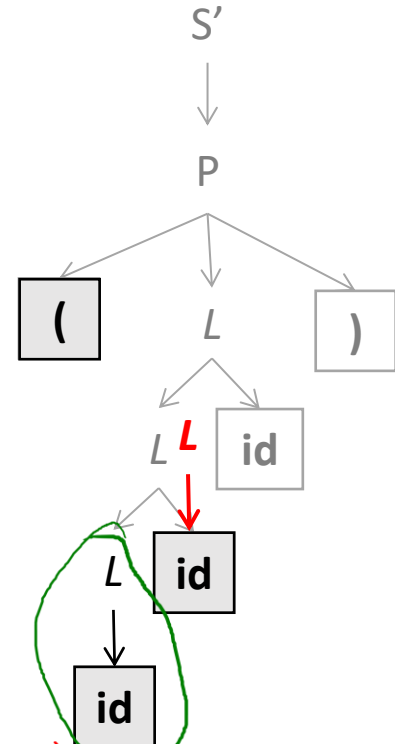
Context: 1 grammar symbols x 1 lookahead

Prefix	Suffix	Action
(	(id id id) eof	shift (
(id	id id id) eof	shift id
(L	id id) eof	Red. ③ $L ::= id$
(L id	id id) eof	shift id
(L L	id) eof	Red. ③ $L ::= L id$
(L L	id) eof	

Grammar G

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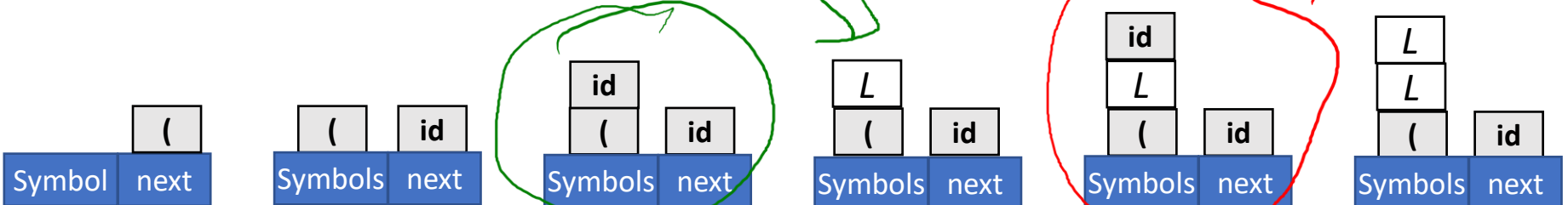
Correct Parse Tree



These are "viable prefixes":
They can reduce to the root

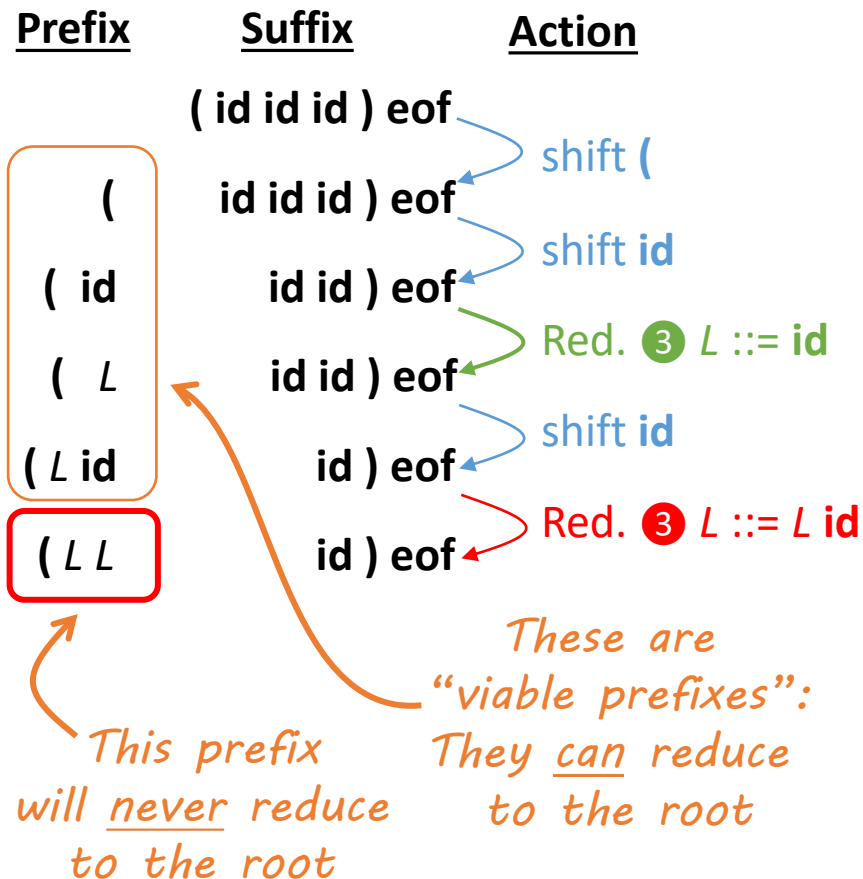
This prefix will never reduce to the root

Naïve / bad idea:



LR Parser – Towards Implementation

LR Parser Construction



The parser must maintain a viable prefix

- Ensures no misstep taken
- How to capture the (infinite) viable prefixes?

AMAZING Fact:

- For a deterministic CFG the viable prefixes form a regular language
 - So there's a DFA for it!

LR Parser – Towards Implementation

LR Parser Construction

Prefix	Suffix	Action
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(L	id id) eof	shift id
(L id	id) eof	Red. ③ $L ::= L id$
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These are “viable prefixes”: They can reduce to the root

This prefix will never reduce to the root

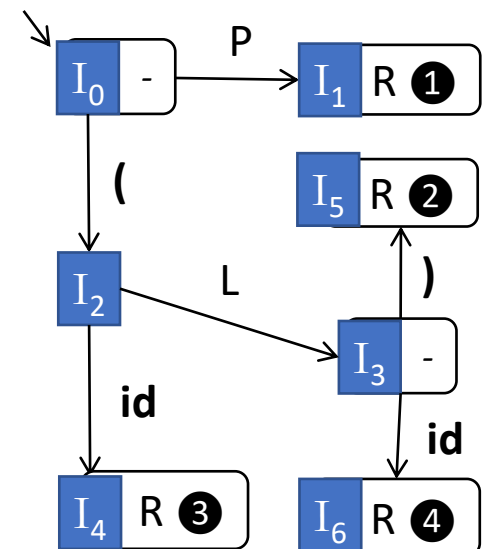
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- $S' ::= P$
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G Parser Automaton



LR Parser – Towards Implementation

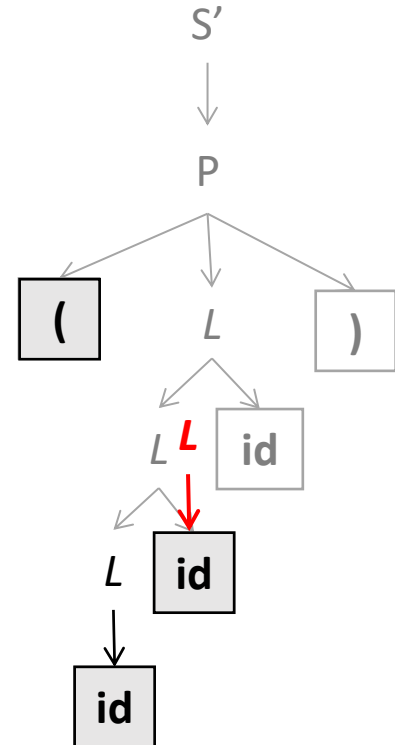
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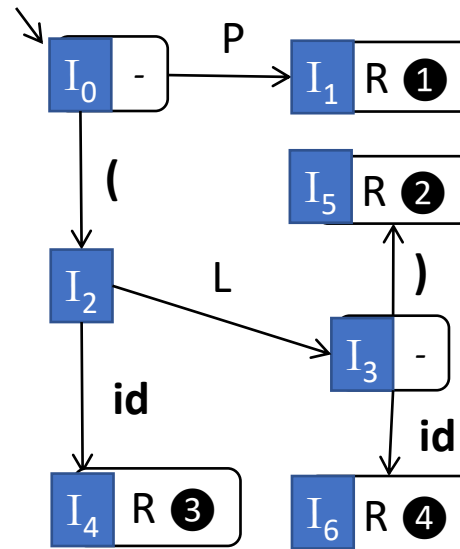
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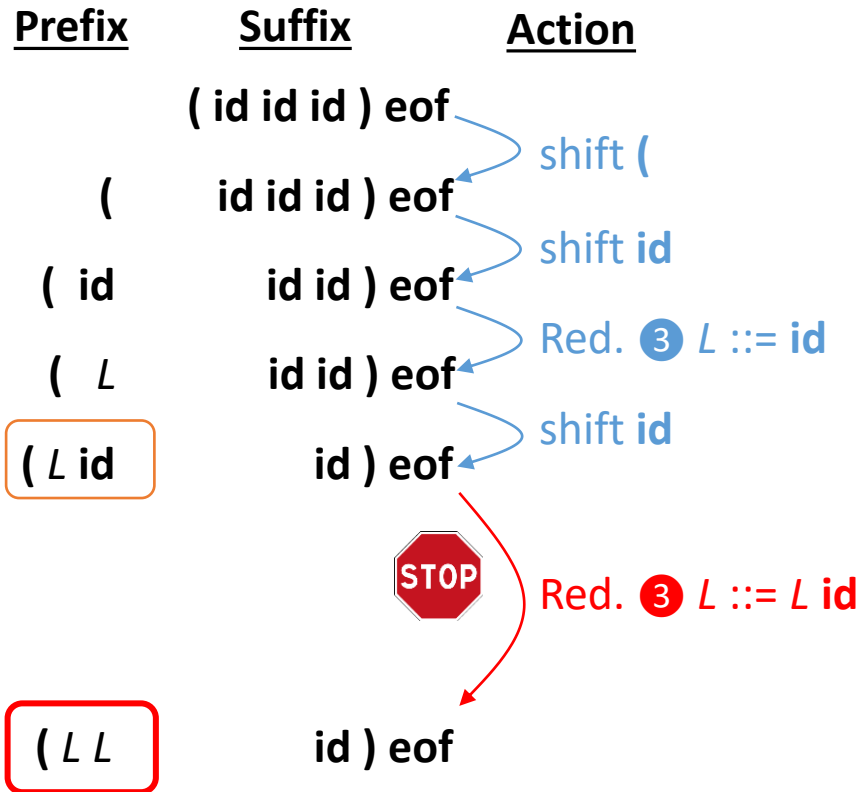


G Parser Automaton



LR Parser – Towards Implementation

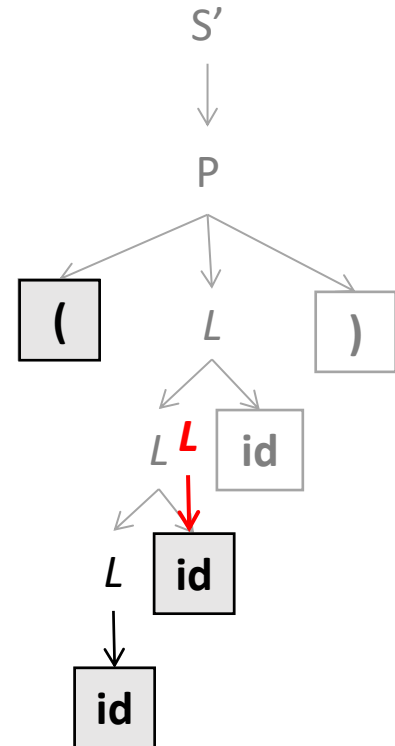
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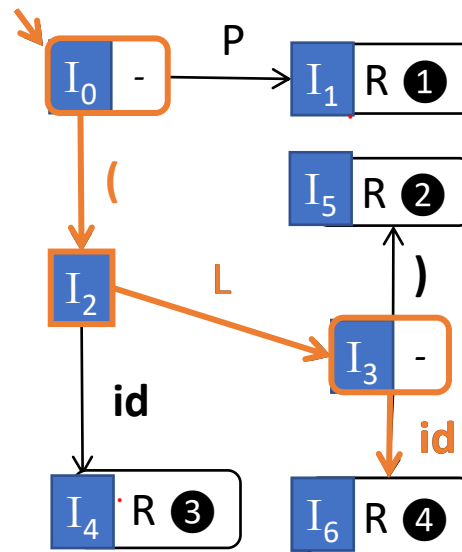
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LR Parser – Towards Implementation

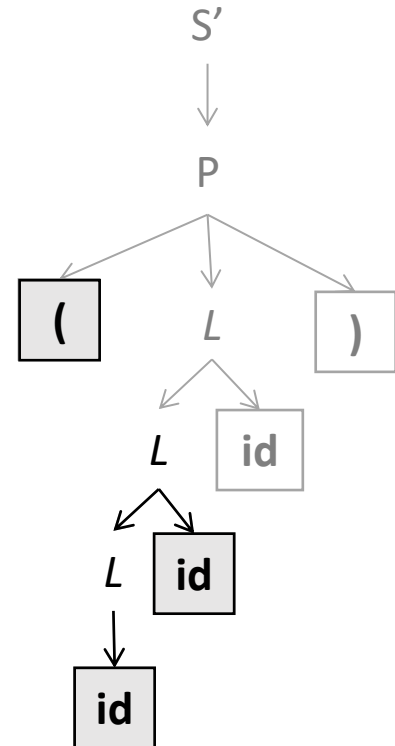
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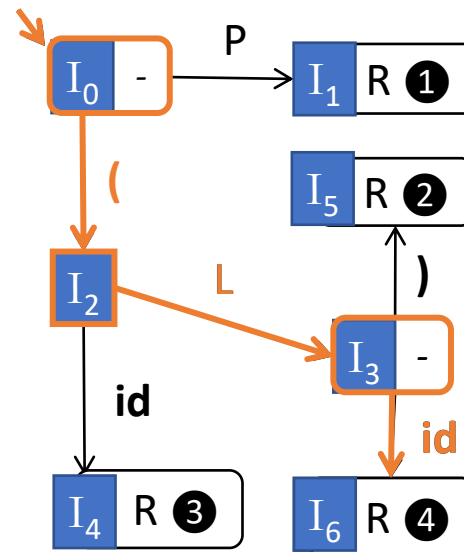
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Correct Parse Tree



G Parser Automaton



Viable Prefix Summary

LR Parser Construction

Any DCFG has a regular language of viable prefixes

- We can use the DFA for that language to prevent missteps

Advancing the automaton

- Terminal Edges – add a terminal to the prefix
- Nonterminal Edges – add nonterminal to the prefix
- Accepting states – modify the existing prefix

Key Idea: Run the DFA for viability

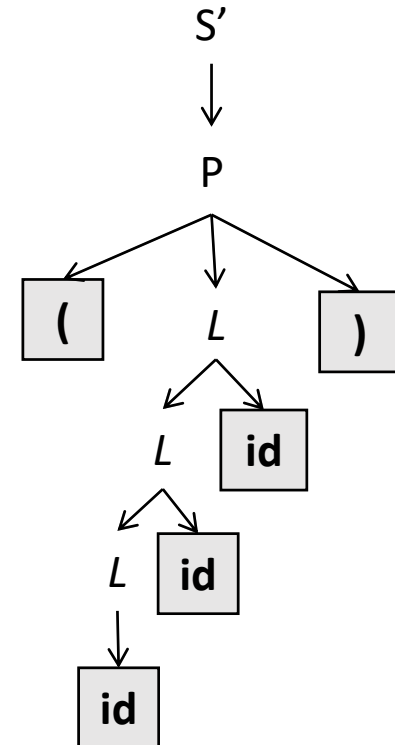
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S'	eof	

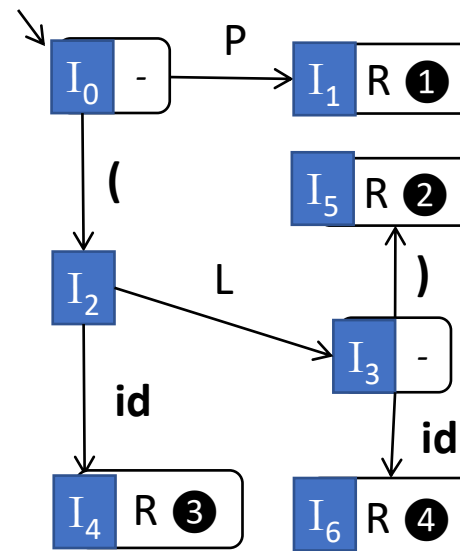
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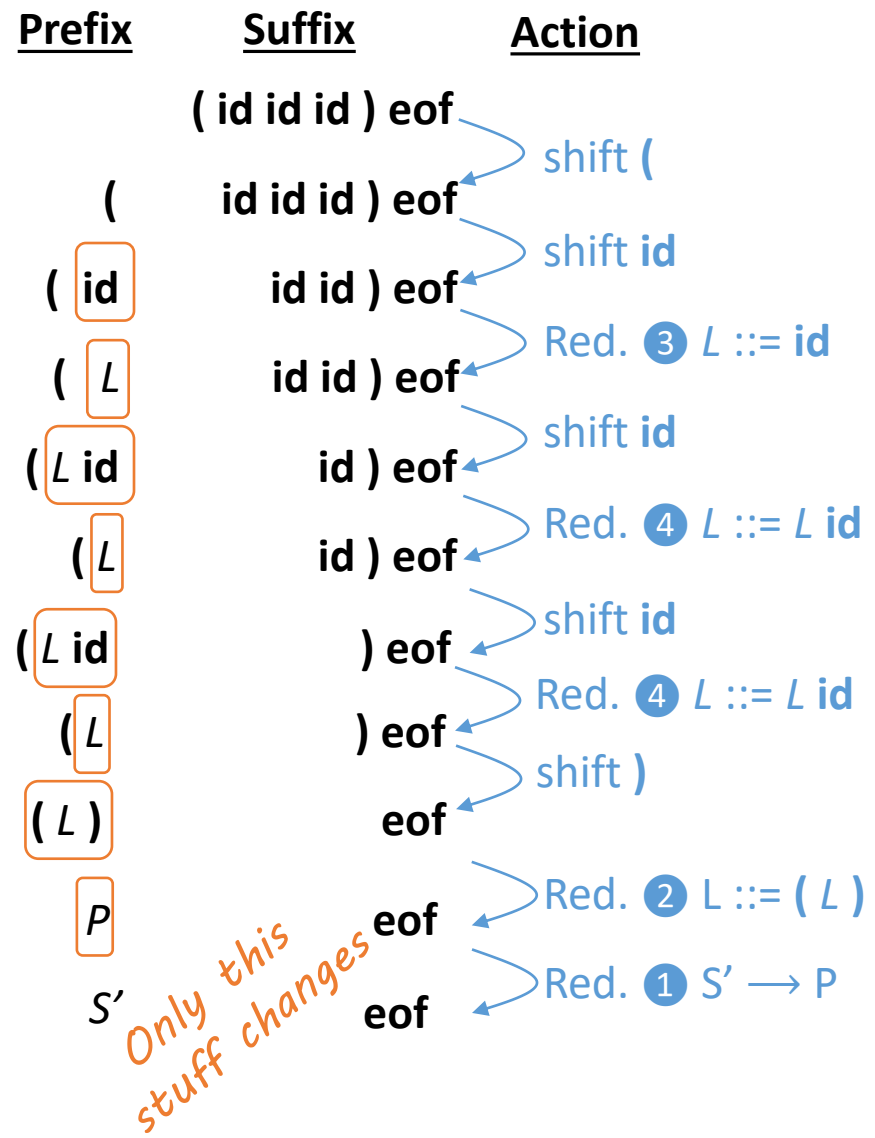


G Parser Automaton

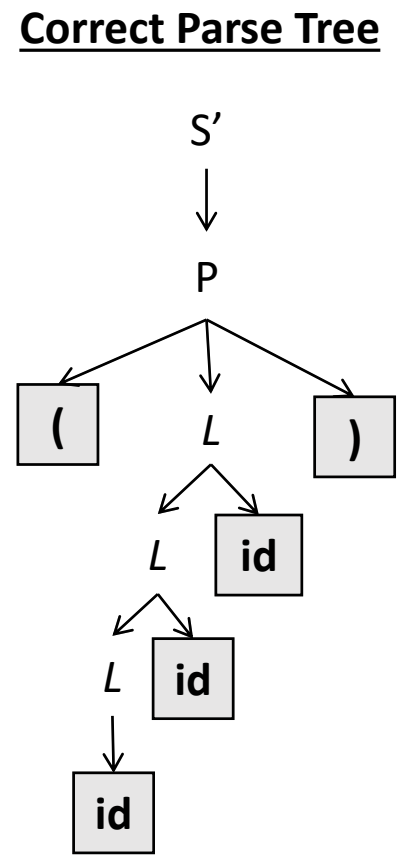
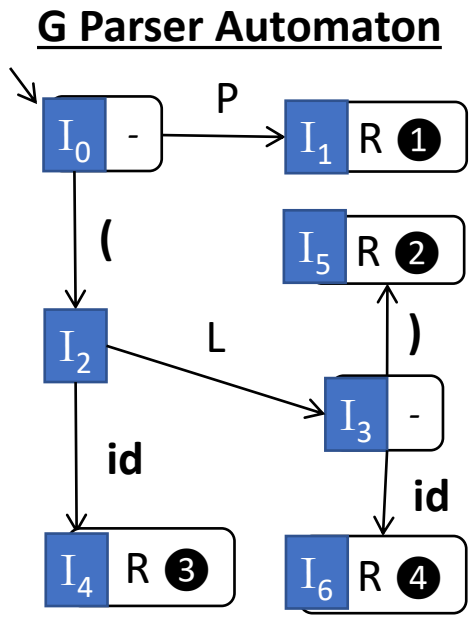


Shortcut: Don't Restart DFA Every Time

LR Parser Construction



- Grammar G**
- ① $S' ::= P$
 - ② $P ::= (L)$
 - ③ $L ::= id$
 - ④ $L ::= L id$



LR Parser Stack

LR Parser Construction

Amazing fact #2: Can keep position in the parse tree *and* position in the automaton in a single stack

- The prefix becomes implicit in the stack
- Can go back to tracking a lookahead “next” token just beyond the prefix

Advancing the automaton

- Terminal Edges – move terminal ~~symbol into prefix~~ Item onto stack
- Nonterminal Edges – add nonterminal ~~symbol to prefix~~
- Accepting states – ~~modify the existing prefix~~ Item onto stack
Pop RHS stack items

LR Parser Stack

LR Parser Construction

Input

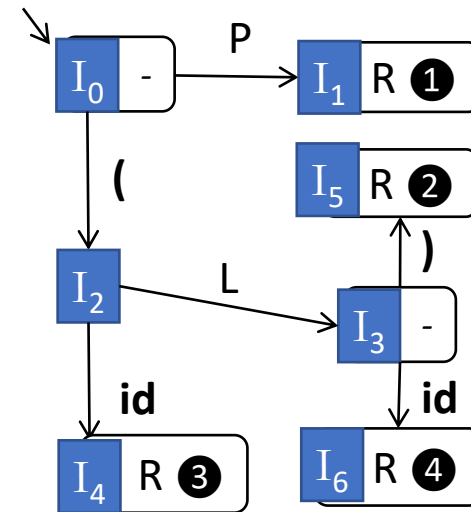
(id id) eof

$T_1 T_2 T_3 T_4 T_5$

Grammar G

- ① $S' ::= P$
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- ③ $L ::= id$
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G Parser Automaton



Terminal edge

$I_0, (\rightarrow I_2$

Terminal edge

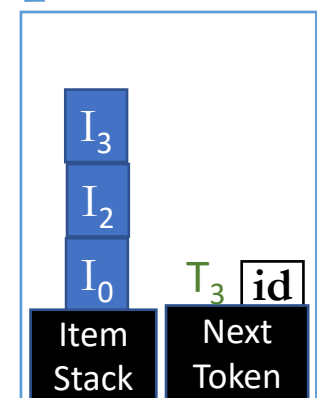
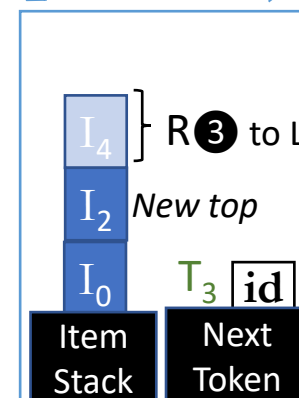
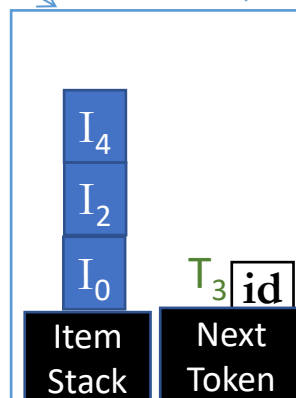
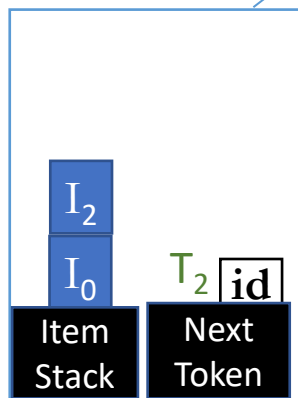
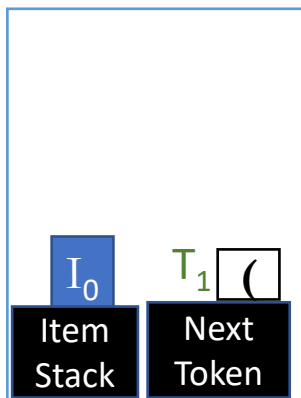
$I_2, id \rightarrow I_4$

Accepting state

Pop 1 stack item

Nonterminal edge

$I_2, L \rightarrow I_3$



LR Parser Stack

LR Parser Construction

Input

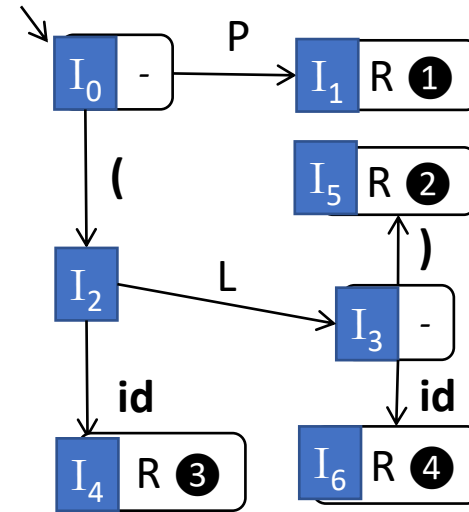
(id id) eof

$T_1 T_2 T_3 T_4 T_5$

Grammar G

- 1 $S' ::= P$
- 2 $P ::= (L)$
- 3 $L ::= id$
- 4 $L ::= L id$

G Parser Automaton



Terminal edge

$I_3, id \rightarrow I_6$

Accepting state 4

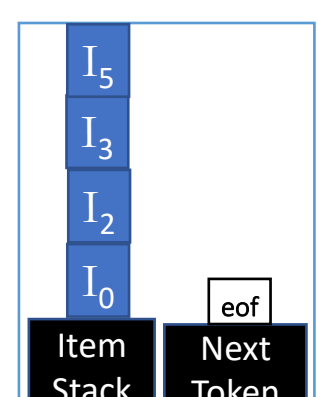
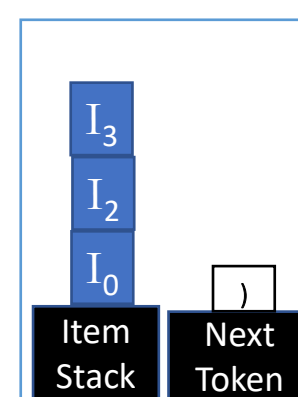
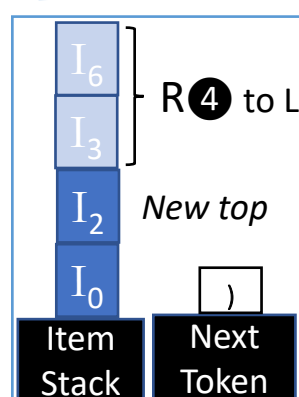
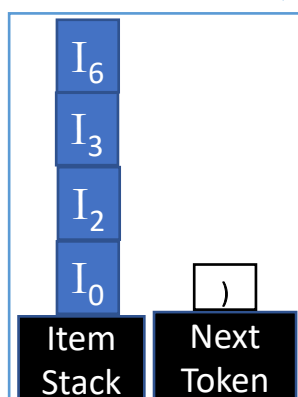
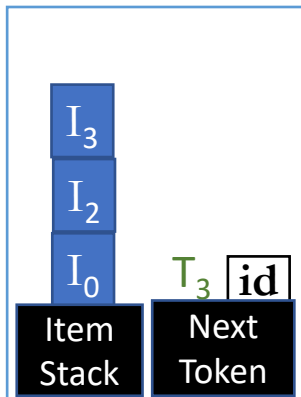
Pop 2 items

~~Terminal edge~~

$I_2, L \rightarrow I_3$

Terminal edge

$I_3,) \rightarrow I_5$



LR Parser Stack

LR Parser Construction

Input

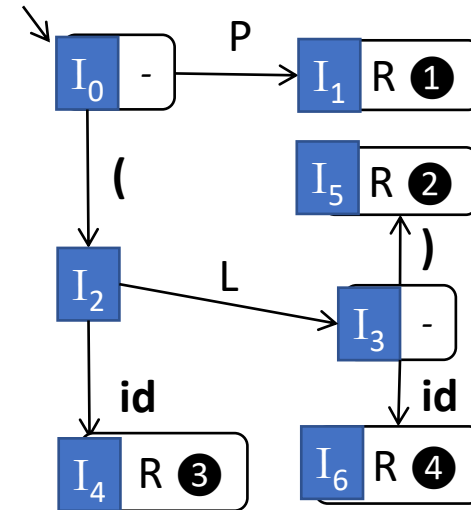
(id id) eof

$T_1 T_2 T_3 T_4 T_5$

Grammar G

- ① $S' ::= P$
- ② $P ::= (L)$
- ③ $L ::= id$
- ④ $L ::= L id$

G Parser Automaton



Accept state ②

Nonterminal edge

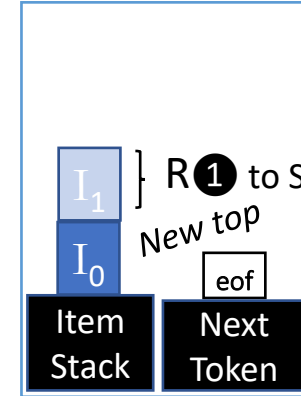
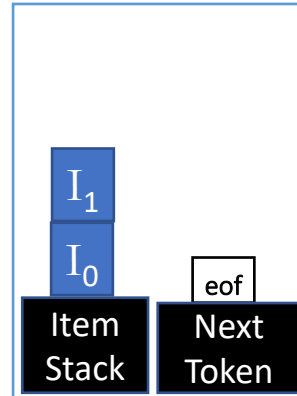
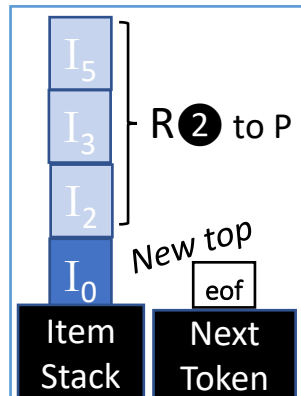
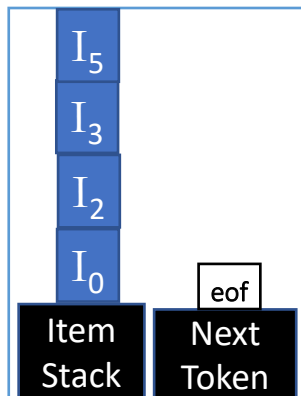
Accept state ①

Pop 3 items

$I_3,) \rightarrow I_5$

Pop 1 item

$I_0, S' \rightarrow \text{Accept!}$



Next: Building the Parser

LR Parser Construction

Final Tasks to complete Implementation:

- Building the “Prefix” Automaton
- Translating the Automaton into a table