Last Time

Review – FIRST Sets

Building LL(1) Parsers

- Transforming grammars:
  - Left factoring
  - Left-recursion elimination
- Building the selector table
  - FIRST Sets

<u>You Should Know</u>

- The intuition behind FIRST and FOLLOW
- The formal definition of FIRST sets

Parsing

Today's Outline
FOLLOW Sets

Building LL(1) Parsers

- LL(1) Game Plan
- Finish up FIRST Sets
- FOLLOW Sets

Parsing

Perspective: Where we’re At

LL(1) Game Plan

Parsers are a bit tricky!

- Sadly, you need to know this to build a compiler frontend

The underlying concepts of FIRST and FOLLOW will be useful for LL(1) and other parsers

- (We’ll talk about 1 other kind – the LR parsers, which is what BISON generates).

What We're Doing: The Big Picture

LL(1) Game Plan

Example Grammar

$$
\begin{aligned}
S ::= &\ (\ S\ ) \\
 |&\ \{\ S\ \} \\
 |&\ \varepsilon
\end{aligned}
$$

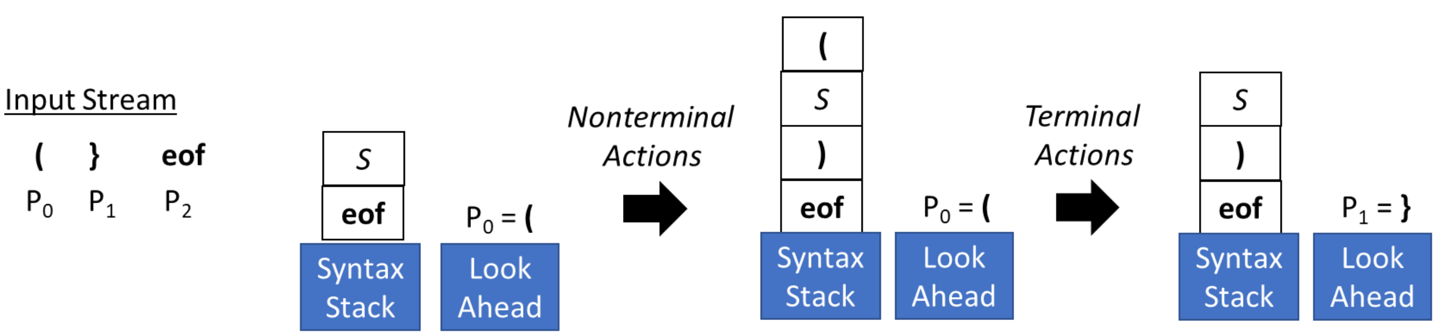
Turning this…

into this…

Selector Table

	(	)	{	}
S	(*S*)	ε	{ *S* }	ε

so we can do this…



What We're Doing: The Big Picture

Building the LL(1) Selector Table

Uses

Uses

Uses

`FIRST(`X`)`	`FIRST(`α`)`	`FOLLOW(`X`)`	`Fill Cells`
Step 1. Build FIRST sets for single symbols of the grammar	Step 2. Build FIRST sets for RHS symbols strings	Step 3. Build FOLLOW sets for Nonterminals	Step 4. Place productions

LL(1) Selector Table Algorithm

Building LL(1) Selector Table

```
for each production X ::= α
   if t is in FIRST(α)
      put X ::= α in Table[X][t]
   if ε is in FIRST(α)
      for each t in FOLLOW(X)
         put X::= α in Table[X][t]
```

We rely on FIRST sets and FOLLOW sets for table construction
But these sets will be useful even beyond the LL parsers

LL(1) Parsers Revisited: Big Picture

LL(1) The Big Picture

Grammar

P_1	S ::= X **b** X
P_2	| **b b**
P_3	X ::= **a** X
P_4	| **c**

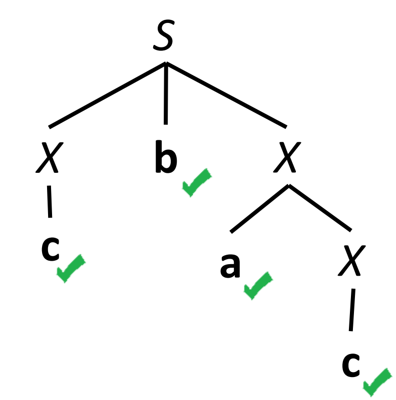
Selector Table

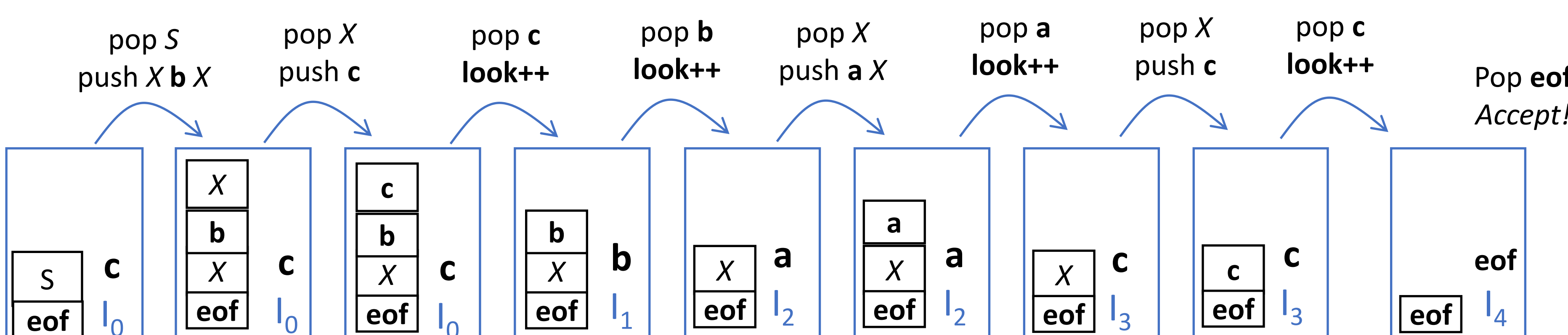
	a	b	c
S	P_1	P_2	P_1
X	P_3		P_4

Token stream

c	b	a	c	eof
l_0	l_1	l_2	l_3	l_4

Predicted Parse Tree





LL(1) Parsers Revisited: Big Picture

LL(1) The Big Picture

Grammar

P_1	S ::= X **b** X
P_2	| **b b**
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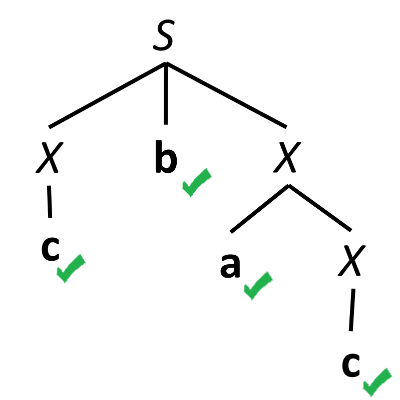


LL(1) Parser "Résumé"

- Goals: to expand the leftmost nonterminal
- Skills: always knows the first leaf of the leftmost nonterminal's subtree

LL(1) Parsers Revisited: Big Picture

LL(1) The Big Picture

LL(1) Parser “Résumé”

- Goals: to expand the leftmost nonterminal
- Skills: always knows the first leaf of the target nonterminal’s subtree

In an LL(1) grammar this is a sufficient skillset!

- Can choose correct production when target’s first leaf token is given *(FIRST sets)*
- Can choose correct production when there is no leaf token based on next subtree over

(FOLLOW sets)

FIRST Set Intuition

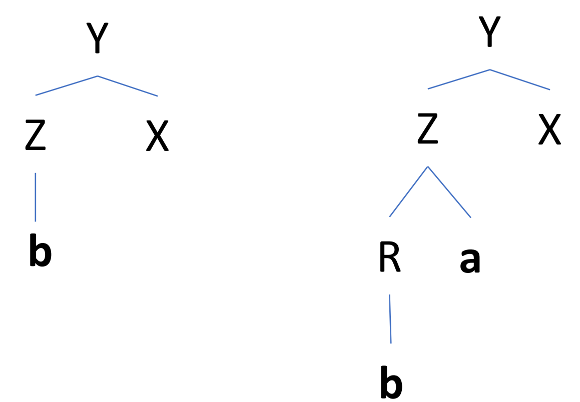
LL(1) The Big Picture

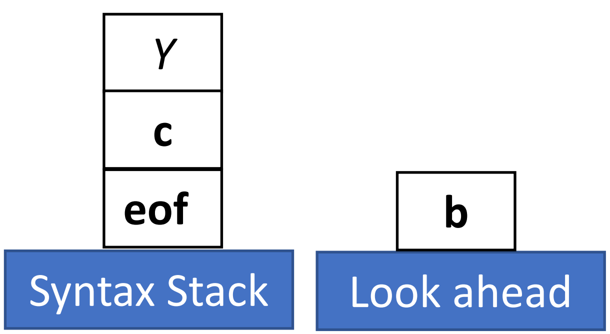
FIRST(X): The set of terminals that begin strings derivable from X, and also, if X can derive ε, then ε is in FIRST(X).

Example Grammar Fragment $\boxed{P_3}$ *Y ::= Z X*

Does P3 apply to this lookahead?

- Yes, if b is in FIRST(Z)





FIRST Set Intuition

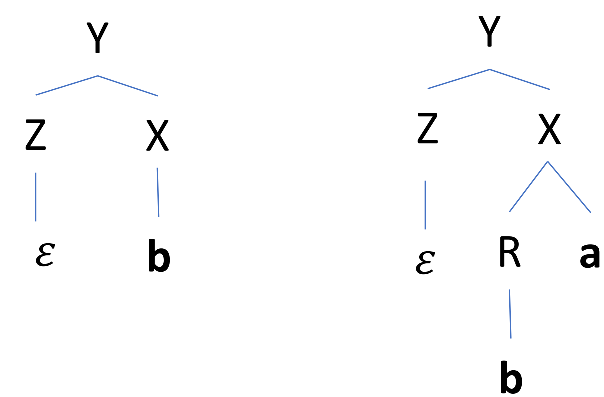
LL(1) The Big Picture

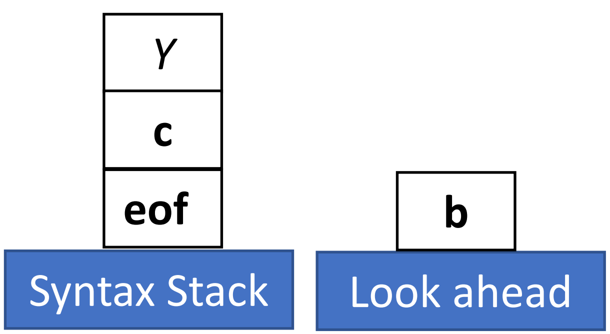
FIRST(X): The set of terminals that begin strings derivable from X, and also, if X can derive ε, then ε is in FIRST(X).

<u>Example Grammar Fragment</u> $\boxed{P_3}$ $Y ::= Z\ X$

Does P3 apply to this lookahead?

- Yes, if b is in FIRST(Z)
- Yes, if ε is in FIRST(Z) and b is in FIRST(X)





FIRST Set Intuition

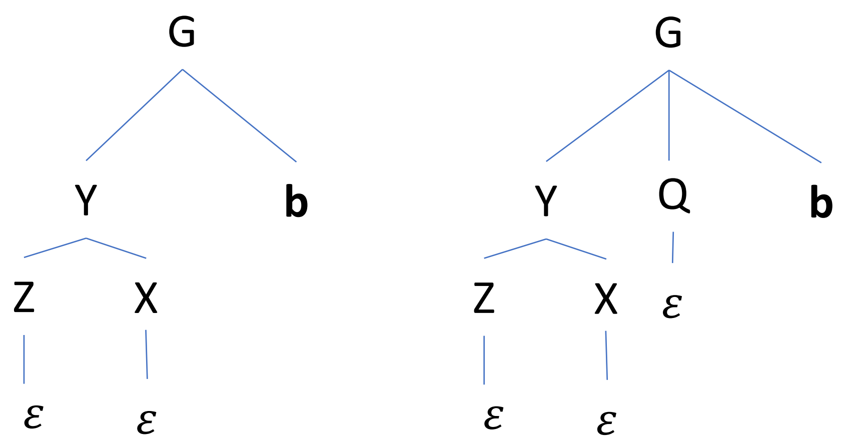
LL(1) The Big Picture

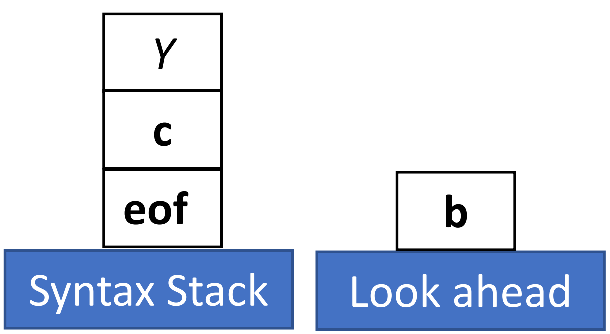
FIRST(X): The set of terminals that begin strings derivable from X, and also, if X can derive ε, then ε is in FIRST(X).

Example Grammar Fragment $\boxed{P_3}$ $Y ::= Z\ X$

Does P3 apply to this lookahead?

- Yes, if b is in FIRST(Z)
- Yes, if ε is in FIRST(Z) and b is in FIRST(X)
- Yes, if ε is in FIRST(Z) and FIRST(X), and b can FOLLOW right after Y





FIRST Set Intuition

LL(1) The Big Picture

FIRST(X): The set of terminals that begin strings derivable from X, and also, if X can derive ε, then ε is in FIRST(X).

Example Grammar Fragment $\boxed{P_3}$ *Y ::= Z X*

Does P3 apply to this lookahead?

- Yes, if b is in FIRST(Z)
- Yes, if *ε* is in FIRST(Z) and b is in FIRST(X)
- Yes, if *ε* is in FIRST(Z) and FIRST(X), and b can FOLLOW right after Y

Syntax Stack
Y
c
eof

Look ahead
b

We're interested in a more general question…

FIRST Set Intuition

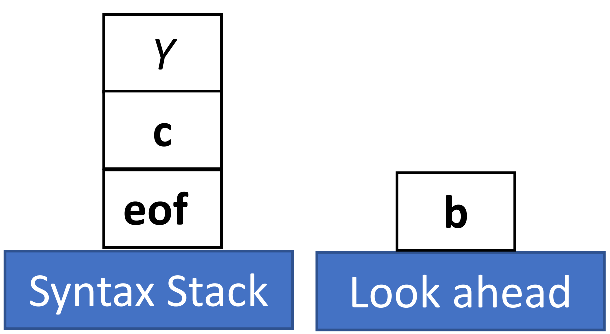
LL(1) The Big Picture

Example Grammar Fragment

P_1	X ::= **a** Y **c**
P_2	| **c**
P_3	Y ::= Z X
P_4	Z ::= **b**
P_5	| **a**

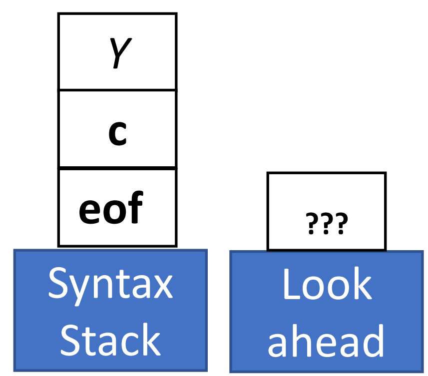
Does P3 apply to this lookahead?

- Yes, if b is in FIRST(Z)
- Yes, if ε is in FIRST(Z) and b is in FIRST(X)
- Yes, if ε is in FIRST(Z) and FIRST(X), and b can FOLLOW right after Y



At what lookahead tokens does P3 apply?

- Those in FIRST(Z)
- If ε is in FIRST(Z), those in FIRST(X)
- If ε is in FIRST(Z) and FIRST(X), those that follow Y



Today’s Outline

FOLLOW Sets

Building LL(1) Parsers

- LL(1) Game Plan
- Building a Grammar’s FIRST sets
- FOLLOW Sets

Parsing

FIRST Sets: Review what we know

Building a Grammar's FIRST Sets

Building FIRST for a terminal t

FIRST(t) = { t }

Building FIRST for ε

FIRST(ε) = { ε }

Building FIRST for a symbol string α

Let α be composed of symbols $\alpha_1\ \alpha_2 \ldots \alpha_n$

- C_1: add FIRST(α_1) - ε
- C_2: For all $k < n$: if $\alpha_1 \ldots \alpha_{k-1}$ is nullable, add FIRST(α_k) - ε
- C_3: If $\alpha_1 \ldots \alpha_n$ is nullable, add ε

Building FIRST for a nonterminal X

For all productions with X on the LHS and $\alpha = \alpha_1\ \alpha_2 \ldots \alpha n$ on the RHS

- C_1: add FIRST(α_1) - ε
- C_2: For all $k < n$: if $\alpha_1 \ldots \alpha_{k-1}$ is nullable, add FIRST(α_k) - ε
- C_3: If $\alpha_1 \ldots \alpha_n$ is nullable, add ε

Same

Building FIRST for a nonterminal X

For all productions with X on the LHS (i.e. X ::= α)

- Add FIRST(α) to FIRST X

FIRST Sets: Review what we know

Building a Grammar's FIRST Sets

Building FIRST for a terminal t

FIRST(t) = { t }

Building FIRST for ε

FIRST(ε) = { ε }

Building FIRST for a symbol string α

Let α be composed of symbols $\alpha_1\ \alpha_2 \ldots \alpha_n$

C_1: add FIRST(α_1) - ε

C_2: For all $k < n$: if $\alpha_1 \ldots \alpha_{k-1}$ is nullable, add FIRST(α_k) - ε

C_3: If $\alpha_1 \ldots \alpha_n$ is nullable, add ε

Mutually recursive (dependency loop)!

This means that there's one additional step we need…

Building FIRST for a nonterminal X

For all productions with X on the LHS (i.e. X ::= α)

Add FIRST(α) to FIRST X

Building FIRST for all Grammar Symbols

<u>For each nonterminal of the grammar</u>

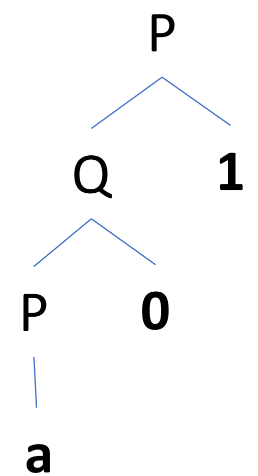
Loop over for all productions (of the form X ::= α, wlog)

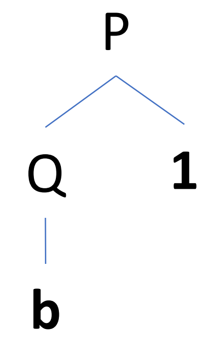
  Add FIRST(α) to FIRST(X)

  (if a set hasn't been computed, use {}, the empty set)

until ***saturation*** (no set changes)

P ::= Q 1
  | a
Q ::= P 0
  | b





FIRST(P) ⊆ FIRST(Q) ⊆ FIRST(P)

Tricks for Computing FIRST Sets

Building Parser Tables

- Begin by computing the single-symbol FIRST sets for each production's LHS
- Run until saturation
- Can help to work bottom-up
- Compute symbol-string FIRST sets for each production's RHS
- Stay hydrated!

S ::= *X* **b** *X*
    | ε
X ::= **a** *X*
    | ε

Today's Outline
FOLLOW Sets

Building LL(1) Parsers

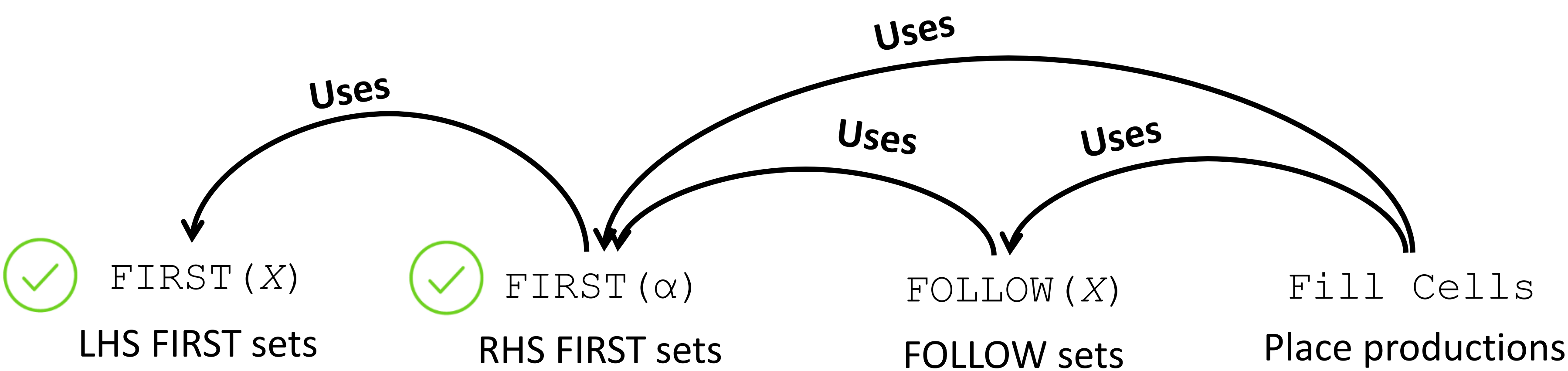
- LL(1) Game Plan
- Building a Grammar's FIRST sets
- FOLLOW Sets

Parsing

Selector Table Dependencies

Building the Selector Table

```
for each production X ::= α
    if t is in FIRST(α)
        put X ::= α in Table[X][t]
    if ε is in FIRST(α)
        for each t in FOLLOW(X)
            put X::= α in Table[X][t]
```



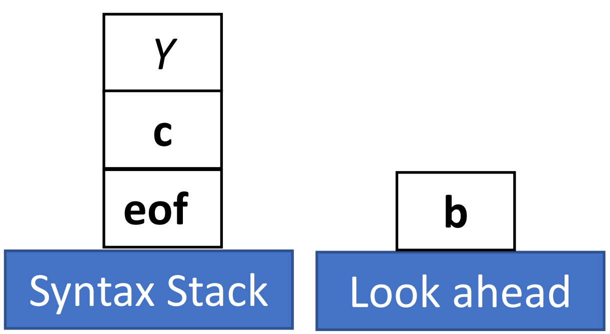
Follow Set Intuition

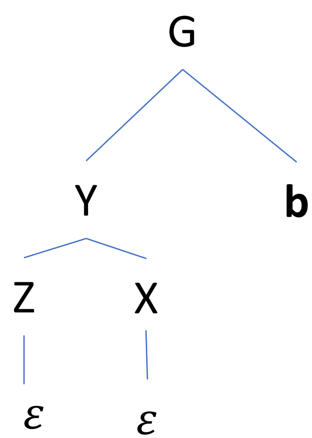
LL(1) The Big Picture

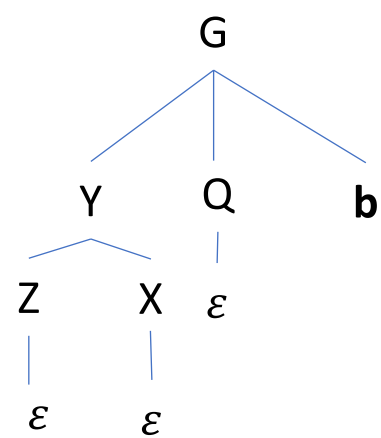
Example Grammar Fragment $\boxed{P_3}$ $Y ::= Z\ X$

Does P3 apply to this lookahead?

- Yes, if b is in FIRST(Z)
- Yes, if ε is in FIRST(Z) and b is in FIRST(X)
- Yes, if ε is in FIRST(Z) and FIRST(X), and b can FOLLOW right after Y



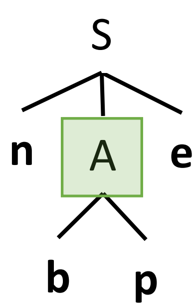




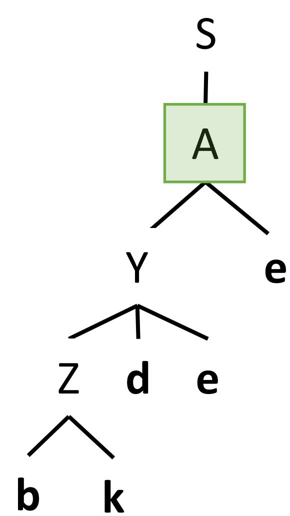
Again, The Parse tree Perspective

Consider the Trees

FIRST(X): The set of terminals that begin strings <u>derivable from</u> X, and also, if X can derive ε, then ε is in FIRST(X).



b ∈ FIRST(A)



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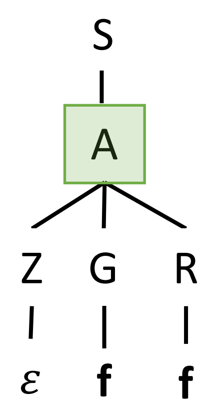
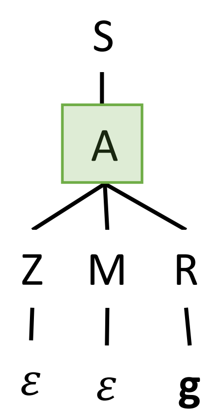
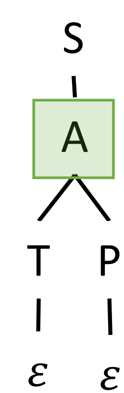


f ∈ FIRST(A)



g ∈ FIRST(A)



ε ∈ FIRST(A)

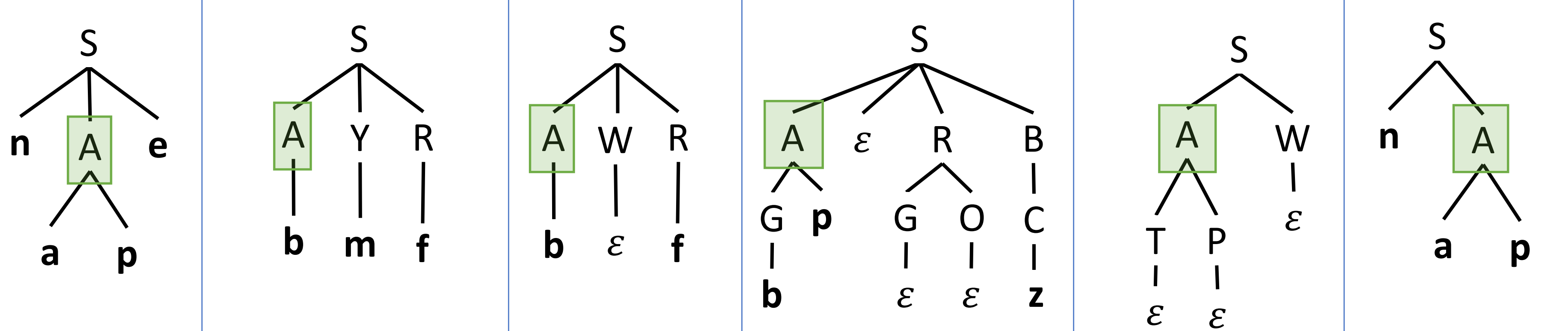
Again, The Parse tree Perspective

Consider the Trees

FIRST(X): The set of terminals that begin strings derivable from X, and also, if X can derive ε, then ε is in FIRST(X).

FOLLOW(X): The set of terminals that begin strings derivable right after X, and **EOF** if there could be *no* terminals after subtree

What does each parse tree say about FOLLOW(A) where S is start?



Yields { **e** }

Yields { **m** }

Yields { **f** }

Yields { **z** }

Yields { **eof** }

Yields { **eof** }

If these were the only parse trees, what is FOLLOW(A)?

{ **e, m, f, z, eof** }

The Importance of FOLLOW

Building Parser Tables

S ::= X **b**
X ::= **a**
X ::= ε

	a	**b**
S	*S ::= X* **b**	*S ::= X* **b**
X		

FIRST(X **b**) = { **a**, **b** }

The Importance of FOLLOW

Building Parser Tables

S ::= X **b**
X ::= **a**
X ::= ε

	a	**b**
S	*S ::= X* **b**	*S::= X* **b**
X	*X* **::= a**	

FIRST(X **b**) = { **a, b** }

FIRST(**a**) = { **a** }

The Importance of FOLLOW

Building Parser Tables

S ::= X **b**
X ::= **a**
$\boxed{X ::= \varepsilon}$

	a	**b**
S	*X* **b**	*X* **b**
X	**a**	ε

FIRST(X **b**) = { **a**, **b** }

FIRST(**a**) = { **a** }

We need to know that
b *follows* X
to place this

The Importance of FOLLOW

Building Parser Tables

S ::= X **b**
X ::= **a**
$\boxed{X ::= \varepsilon}$

	a	**b**
S	*X* **b**	*X* **b**
X	**a**	ε

FIRST(X **b**) = { **a**, **b** }

FIRST(**a**) = { **a** }

We need to know that
b *follows* X
to place this

S ::= X
X ::= **a** *X*
X ::= ε

	a	**EOF**
S	*X*	*X*
X	**a** *X*	ε

FOLLOW Sets, Formally

Building Parser Tables

Those terminals *derivable* *immediately after X*

$$\text{FOLLOW}({\color{red}X}) = \left\{ t \;\middle|\; \left(t \in \Sigma \land S \overset{+}{\Rightarrow} \alpha {\color{red}X} \mathbf{t} \beta \right) \lor \left(t = \mathbf{eof} \land S \overset{+}{\Rightarrow} \alpha {\color{red}X} \right) \right\}$$

also eof *when X ends a derivation*

Example: Building Follow Sets

Building Parser Tables

FOLLOW(*X*) for each nonterminal X

C_1: If *X* is the start nonterminal, add **eof**

For all $Z ::= \alpha\ X\ \beta$ (where α and/or β may be empty)

C_2: Add FIRST(β) – {ε}

C_3: If ε is in FIRST(β) add FOLLOW(*Z*)

C_4: If β is empty add FOLLOW(*Z*)

Repeat for each nonterminal until saturation

Grammar

1. S ::= **a**
2. S ::= **b** R
3. Q ::= ε
4. R ::= Q **c**
5. R ::= Q S
6. R ::= Q Q

FOLLOW(*X*) for nonterminal *X*

C_1: If *X* is the start nonterminal, add **eof**

For all Z ::= α *X* β (where α and/or β may be empty)

- C_2: Add FIRST(β) – {ε}
- C_3: If ε is in FIRST(β) add FOLLOW(Z)
- C_4: If β is empty add FOLLOW(Z)

Repeat for each nonterminal until saturation

FIRST(S) = { a, b }

FIRST(Q) = { ε }

FIRST(R) = { **c**, **a**, **b**, ε}

FIRST(Q **c**) = { c }

FIRST(Q S) = { a, b }

FIRST(Q Q) = { ε }

FOLLOW(S) = { **eof** }

FOLLOW(Q)

FOLLOW(R)

Building Follow(S) (S in for X)

C_1: S is the start nonterminal, so add **eof**

Rules of the form Z ::= α *X* β

R Q S empty

R ::= *Q* $\boxed{S}$

C_2: β is empty, so add nothing

C_3: β is empty, so N/A

C_4: β is empty, so add FOLLOW(R), which is currently nothing

Grammar

❶ S ::= **a**

❷ S ::= **b** R

❸ Q ::= ε

❹ R ::= Q ***c***

❺ R ::= Q S

❻ R ::= Q Q

FOLLOW(*X*) for nonterminal *X*

C_1: If X is the start nonterminal, add **eof**

For all Z ::= α X β (where α and/or β may be empty)

- C_2: Add FIRST(β) – {ε}
- C_3: If ε is in FIRST(β) add FOLLOW(Z)
- C_4: If β is empty add FOLLOW(Z)

Repeat for each nonterminal until saturation

FIRST(S) = { a, b }

FIRST(Q) = { ε }

FIRST(R) = { **c**, **a**, **b**, ε}

FIRST(Q **c**) = { c }

FIRST(Q S) = { a, b }

FIRST(Q Q) = { ε }

FOLLOW(S) = { **eof** }

FOLLOW(Q)

FOLLOW(R)

Building Follow(Q) (Q in for X)

C_1: N/A (Q not the start nonterminal)

Rules of the form Z ::= α X β

R empty Q c

R ::= $\boxed{Q}$ **c** adds { **c** }

R ::= $\boxed{Q}$ S

R ::= $\boxed{Q}$ Q

R ::= Q $\boxed{Q}$

C_2: β is **c**, add FIRST(**c**) - ε = { **c** }

C_3: β is **c**, $\varepsilon \notin$ FIRST(**c**), so N/A

C_4: β is not empty, so N/A

Grammar

1. $S ::= \mathbf{a}$
2. $S ::= \mathbf{b}\ R$
3. $Q ::= \varepsilon$
4. $R ::= Q\ \mathbf{c}$
5. $R ::= Q\ S$
6. $R ::= Q\ Q$

FOLLOW(X) for nonterminal X

C_1: If X is the start nonterminal, add **eof**

For all $Z ::= \alpha\ X\ \beta$ (where α and/or β may be empty)

- C_2: Add FIRST(β) – {ε}
- C_3: If ε is in FIRST(β) add FOLLOW(Z)
- C_4: If β is empty add FOLLOW(Z)

Repeat for each nonterminal until saturation

FIRST(S) = { a, b }

FIRST(Q) = { ε }

FIRST(R) = { **c**, **a**, **b**, ε}

FIRST(Q **c**) = { c }

FIRST(Q S) = { a, b }

FIRST(Q Q) = { ε }

FOLLOW(S) = { **eof** }

FOLLOW(Q)

FOLLOW(R)

Building Follow(Q) (Q in for X)

C_1: N/A (Q not the start nonterminal)

Rules of the form $Z ::= \alpha\ X\ \beta$

R empty Q S

R ::= $\boxed{Q}$ **c** adds { **c** }

R ::= $\boxed{Q}$ S adds {**a**,**b**}

R ::= $\boxed{Q}$ Q

R ::= Q $\boxed{Q}$

C_2: β is S, FIRST(S) - ε = { **a**, **b** }

C_3: β is S, $\varepsilon \notin$ FIRST(S), so N/A

C_4: β is not empty, so N/A

Grammar

❶ $S ::= \mathbf{a}$

❷ $S ::= \mathbf{b}\ R$

❸ $Q ::= \varepsilon$

❹ $R ::= Q\ \mathbf{c}$

❺ $R ::= Q\ S$

❻ $R ::= Q\ Q$

FOLLOW(*X*) for nonterminal *X*

C_1: If *X* is the start nonterminal, add **eof**

For all $Z ::= \alpha\ X\ \beta$ (where α and/or β may be empty)

- C_2: Add FIRST(β) – {ε}
- C_3: If ε is in FIRST(β) add FOLLOW(*Z*)
- C_4: If β is empty add FOLLOW(*Z*)

Repeat for each nonterminal until saturation

FIRST(S) = { a, b }

FIRST(Q) = { ε }

FIRST(R) = { **c**, **a**, **b**, ε}

FIRST(Q **c**) = { c }

FIRST(Q S) = { a, b }

FIRST(Q Q) = { ε }

FOLLOW(S) = { **eof** }

FOLLOW(Q)

FOLLOW(R)

Building Follow(Q) (Q in for X)

C_1: N/A (*Q* not the start nonterminal)

Rules of the form Z ::= α *X* β

R empty Q Q

R ::= $\boxed{Q}$ **c** adds { **c** }

R ::= $\boxed{Q}$ S adds {**a**,**b**}

R ::= $\boxed{Q}$ *Q* adds { }

R ::= *Q* $\boxed{Q}$ adds { }

C_2: β is *Q*, FIRST(*Q*) - ε = { }

C_3: β is *Q*, Z is R, $\varepsilon \in$ FIRST(Q), add FOLLOW(R) = { }

C_4: β is not empty, so N/A

Grammar

1. *S* ::= **a**
2. *S* ::= **b** *R*
3. *Q* ::= ε
4. *R* ::= *Q* **c**
5. *R* ::= *Q S*
6. *R* ::= *Q Q*

FOLLOW(*X*) for nonterminal *X*

C_1: If *X* is the start nonterminal, add **eof**

For all *Z* ::= α *X* β (where α and/or β may be empty)

- C_2: Add FIRST(β) – {ε}
- C_3: If ε is in FIRST(β) add FOLLOW(*Z*)
- C_4: If β is empty add FOLLOW(*Z*)

Repeat for each nonterminal until saturation

FIRST(S) = { a, b }

FIRST(Q) = { ε }

FIRST(R) = { **c**, **a**, **b**, ε}

FIRST(Q **c**) = { c }

FIRST(Q S) = { a, b }

FIRST(Q Q) = { ε }

FOLLOW(S) = { **eof** }

FOLLOW(Q)

FOLLOW(R)

Building Follow(Q) (Q in for X)

C_1: N/A (*Q* not the start nonterminal)

Rules of the form Z ::= α *X* β

R Q Q empty

R ::= $\boxed{Q}$ **c** adds { **c** }

R ::= $\boxed{Q}$ S adds {**a**,**b**}

R ::= $\boxed{Q}$ *Q* adds { }

R ::= *Q* $\boxed{Q}$ adds { }

C_2: β is empty, so add { }

C_3: β is empty, so N/A

C_4: β is not empty, Z is R,
add FOLLOW(R) = { }

Grammar

❶ S ::= **a**

❷ S ::= **b** R

❸ Q ::= ε

❹ R ::= Q ***c***

❺ R ::= Q S

❻ R ::= Q Q

FOLLOW(X) for nonterminal X

C_1: If X is the start nonterminal, add **eof**

For all Z ::= α X β (where α and/or β may be empty)

C_2: Add FIRST(β) – {ε}

C_3: If ε is in FIRST(β) add FOLLOW(Z)

C_4: If β is empty add FOLLOW(Z)

Repeat for each nonterminal until saturation

FIRST(S) = { a, b }

FIRST(Q) = { ε }

FIRST(R) = { **c**, **a**, **b**, ε}

FIRST(Q **c**) = { C }

FIRST(Q S) = { a, b }

FIRST(Q Q) = { ε }

FOLLOW(S) = { **eof** }

FOLLOW(Q) = { **c**, **a**, **b** }

FOLLOW(R)

Building Follow(Q) (Q in for X)

C_1: N/A (Q not the start nonterminal)

Rules of the form Z ::= α X β

R ::= $\boxed{Q}$ **c** adds { **c** }

R ::= $\boxed{Q}$ S adds {**a**,**b**}

R ::= $\boxed{Q}$ Q adds { }

R ::= Q $\boxed{Q}$ adds { }

Grammar

❶ S ::= **a**

❷ S ::= **b** R

❸ Q ::= ε

❹ R ::= Q **c**

❺ R ::= Q S

❻ R ::= Q Q

FOLLOW(X) for nonterminal X

C_1: If X is the start nonterminal, add **eof**

For all Z ::= α X β (where α and/or β may be empty)

C_2: Add FIRST(β) – {ε}

C_3: If ε is in FIRST(β) add FOLLOW(Z)

C_4: If β is empty add FOLLOW(Z)

Repeat for each nonterminal until saturation

FIRST(S) = { a, b }

FIRST(Q) = { ε }

FIRST(R) = { **c**, **a**, **b**, ε}

FIRST(Q **c**) = { c }

FIRST(Q S) = { a, b }

FIRST(Q Q) = { ε }

FOLLOW(S) = { **eof** }

FOLLOW(Q) = { **c**, **a**, **b** }

FOLLOW(R)

Building Follow(R) (R in for X)

C_1: N/A (R not the start nonterminal)

Rules of the form Z ::= α X β

S b R empty

S ::= **b** $\boxed{R}$ adds { **eof** }

C_2: β is empty, add { }

C_3: β is empty, N/A

C_4: Z is S, add FOLLOW(S) = { **eof** }

Grammar

1. $S ::= \mathbf{a}$
2. $S ::= \mathbf{b}\ R$
3. $Q ::= \varepsilon$
4. $R ::= Q\ \mathbf{c}$
5. $R ::= Q\ S$
6. $R ::= Q\ Q$

FOLLOW(*X*) for nonterminal *X*

C_1: If *X* is the start nonterminal, add **eof**

For all $Z ::= \alpha\ X\ \beta$ (where α and/or β may be empty)

- C_2: Add FIRST(β) – {ε}
- C_3: If ε is in FIRST(β) add FOLLOW(*Z*)
- C_4: If β is empty add FOLLOW(*Z*)

Repeat for each nonterminal until saturation

FIRST(S) = { a, b }

FIRST(Q) = { ε }

FIRST(R) = { **c**, **a**, **b**, ε}

FIRST(Q **c**) = { c }

S ::= **b** $\boxed{R}$ adds { **eof** }

FIRST(Q S) = { a, b }

FIRST(Q Q) = { ε }

FOLLOW(S) = { **eof** }

FOLLOW(Q) = { **c**, **a**, **b** }

FOLLOW(R) = { **eof** }

Grammar

1. S ::= **a**
2. S ::= **b** R
3. Q ::= ε
4. R ::= Q **c**
5. R ::= Q S
6. R ::= Q Q

FOLLOW(X) for nonterminal X

C_1: If X is the start nonterminal, add **eof**

For all $Z ::= \alpha\ X\ \beta$ (where α and/or β may be empty)

C_2: Add FIRST(β) – {ε}

C_3: If ε is in FIRST(β) add FOLLOW(Z)

C_4: If β is empty add FOLLOW(Z)

Repeat for each nonterminal $\boxed{\text{until saturation}}$

FIRST(S) = { a, b }

FIRST(Q) = { ε }

FIRST(R) = { **c**, **a**, **b**, ε}

FIRST(Q **c**) = { c }

FIRST(Q S) = { a, b }

FIRST(Q Q) = { ε }

FOLLOW(S) = { **eof** }

FOLLOW(Q) = { **c**, **a**, **b** }

FOLLOW(R) = { **eof** }

All done?

Grammar

❶ S ::= **a**

❷ S ::= **b** R

❸ Q ::=

❹ R

FOLLOW(X) for nonterm

C_1: If X is the start n

For all Z ::= α X β (w

C_2: Add FIRST(β)

Recall computing FOLLOW(Q)

We used a set that later changed!

in for

nonterminal)

m Z ::= α X β

R Q Q empty

FIR

FIRST(Q S) = {

FIRST(Q Q) = { ε

FOLLOW(S) = { **eof** }

FOLLOW(Q)

FOLLOW(R)

R ::= Q S adds {**a**,**b**}

R ::= Q Q adds { }

R ::= Q Q adds { }

C_2: β is empty, so add { }

C_3: β is empty, so N/A

C_4: β is not empty, Z is R, add FOLLOW(R) = { }

Grammar

❶ S ::= **a**

❷ S ::= **b** R

❸ Q ::= ε

❹ R ::= Q **c**

❺ R ::= Q S

❻ R ::= Q Q

FOLLOW(*X*) for nonterminal *X*

C_1: If X is the start nonterminal, add **eof**

For all Z ::= α X β (where α and/or β may be empty)

C_2: Add FIRST(β) – {ε}

C_3: If ε is in FIRST(β) add FOLLOW(Z)

C_4: If β is empty add FOLLOW(Z)

Repeat for each nonterminal until saturation

FIRST(S) = { a, b }

FIRST(Q) = { ε }

FIRST(R) = { **c**, **a**, **b**, ε}

FIRST(Q **c**) = { c }

FIRST(Q S) = { a, b }

FIRST(Q Q) = { ε }

FOLLOW(S) = { **eof** }

FOLLOW(Q) = { **c, a, b** }

FOLLOW(R) = { **eof** }

PSA

Run FOLLOW and FIRST computations until saturation

Round 2

FOLLOW(S) = { **eof }**

FOLLOW(Q) = { **c, a, b, eof** }

FOLLOW(R) = { **eof** }

Round 3

FOLLOW(S) = { **eof** }

FOLLOW(Q) = { **c, a, b, eof** }

FOLLOW(R) = { **eof** }

Review: Set Dependencies

Building LL(!) Selector Table

Uses

Uses

Uses

`FIRST(X)`

Step 1.

Build FIRST sets for single symbols of the grammar

`FIRST(α)`

Step 2.

Build FIRST sets for RHS symbols strings

`FOLLOW(X)`

Step 3.

Build FOLLOW sets for Nonterminals

`Fill Cells`

Step 4.

Place productions

LL(1) Selector Table Algorithm

Building LL(1) Selector Table

```
for each production X ::= α
    for each terminal t in FIRST(α)
        put X ::= α in Table[X][t]
    if ε is in FIRST(α)
        for each t in FOLLOW(X)
            put X::= α in Table[X][t]
```

LL(1) Selector Table Algorithm

Building LL(1) Selector Table

Time permitting: Examples

Table[X][t]	CFG

<pre>for each production X ::= α
 for each terminal **t** in FIRST(α)
   put X ::= α in Table[X][**t**]
 if ε is in FIRST(α)
   for each terminal **t** in FOLLOW(X)
     put X ::= α in Table[X][**t**]</pre>
	S ::= B **c** | D B
	B ::= **a b** | **c** S
	D ::= **d** | ε

FIRST (S) = { **a, c, d** }
FIRST (B) = { **a, c** }
FIRST (D) = { **d**, ε }
FIRST (B **c**) = { **a, c** }
FIRST (D B) = { **d, a, c** }
FIRST (**a b**) = { **a** }
FIRST (**c** *S*) = { **c** }

FOLLOW (S) = { **eof, c** }
FOLLOW (B) = { **c, eof** }
FOLLOW (D) = { **a, c** }

	a	**b**	**c**	**d**	**eof**
S					
B	**a b**				
D					

For each production X ::= α

B ::= **a b**   *B* **a b**

Look at terminals in FIRST(α) = { **a** }:

Put B ::= **a b** @ Table[*B*][**a**]

ε is not in FIRST(α) = { **a** }:

Done with this production

Table[X][t]

```
for each production X ::= α
 for each terminal t in FIRST(α)
   put X ::= α in Table[X][t]
 if ε is in FIRST(α)
   for each terminal t in FOLLOW(X)
     put X ::= α in Table[X][t]
```

CFG

S ::= B **c** | D B

B ::= **a b** | **c** S

D ::= **d** | ε

FIRST (S) = { **a, c, d** }
FIRST (B) = { **a, c** }
FIRST (D) = { **d**, ε }
FIRST (B **c**) = { **a, c** }
FIRST (D B) = { **d, a, c** }
FIRST (**a b**) = { **a** }
FIRST (**c** *S*) = { **c** }

FOLLOW (S) = { **eof, c** }
FOLLOW (B) = { **c**, **eof** }
FOLLOW (D) = { **a, c** }

	a	**b**	**c**	**d**	**eof**
S					
B	**a b**				
D	ε		ε		

For each production X ::= α

$D ::= \varepsilon$ D ε

Look at terminals in FIRST(α) = { ε }

There are none

Because ε is in FIRST(α)

Look at everything in Follow(X) = { **a**, **c** }

Put D ::= ε @ Table[*D*][**a**]

Put D ::= ε @ Table[*D*][**c**]

Table[X][t]

```
for each production X ::= α
 for each terminal t in FIRST(α)
   put X ::= α in Table[X][t]
 if ε is in FIRST(α)
   for each terminal t in FOLLOW(X)
     put X ::= α in Table[X][t]
```

CFG

S ::= B **c** | D B

B ::= **a b** | **c** S

D ::= **d** | ε

FIRST (S) = { **a, c, d** }
FIRST (B) = { **a, c** }
FIRST (D) = { **d**, ε }
FIRST (B **c**) = { **a, c** }
FIRST (D B) = { **d, a, c** }
FIRST (**a b**) = { **a** }
FIRST (**c** *S*) = { **c** }

FOLLOW (S) = { **eof, c** }
FOLLOW (B) = { **c, eof** }
FOLLOW (D) = { **a, c** }

	a	**b**	**c**	**d**	**eof**
S	*D B*		*D B*		
B	**a b**				
D	ε		ε		

For each production X ::= α

S ::= D B   *S*   *D B*

Look at terminals in FIRST(α) = { **d, a, c** }

Put S ::= D B @ Table[S][**d**]

Put S ::= D B @ Table[S][**a**]

Put S ::= D B @ Table[S][**c**]

ε is not in FIRST(α) = { **d, a, c** }:

Done with this production

Table[X][t]

```
for each production X ::= α
 for each terminal t in FIRST(α)
   put X ::= α in Table[X][t]
 if ε is in FIRST(α)
   for each terminal t in FOLLOW(X)
     put X ::= α in Table[X][t]
```

CFG

S ::= B **c** | D B

B ::= **a b** | **c** S

D ::= **d** | ε

FIRST (S) = { **a**, **c**, **d** }
FIRST (B) = { **a**, **c** }
FIRST (D) = { **d**, ε }
FIRST (B **c**) = { **a**, **c** }
FIRST (D B) = { **d**, **a**, **c** }
FIRST (**a b**) = { **a** }
FIRST (**c** *S*) = { **c** }

FOLLOW (S) = { **eof**, **c** }
FOLLOW (B) = { **c**, **eof** }
FOLLOW (D) = { **a**, **c** }

	a	**b**	**c**	**d**	**eof**
S	*D B* *B* **c**		*D B* *B* **c**		
B	**a b**				
D	ε		ε		

For each production X ::= α

S ::= B **c** *S* *B* **c**

Look at terminals in FIRST(α) = { **a**, **c** }

Put S ::= B C @ Table[S][**a**]

Put S ::= B C @ Table[S][**c**]

ε is not in FIRST(α) = { **a** }:

Done with this production

Table[X][t]

```
for each production X ::= α
  for each                in FIRST(α)
    put               e[X][t]
  if ε i
    for e               t in FOLLOW(X)
      put               able[X][t]
```

Collision! Grammar is not LL(1)

CFG

S ::= B **c** | D B

B ::= **a b** | **c** S

D ::= **d** | ε

FIRST (S) = { **a, c, d** }

FIRST (B) = { **a, c** }

FIRST (D) = { **d**, ε }

FIRST (B **c**) = { **a, c** }

FIRST (D B) = { **d, a, c** }

FIRST (**a b**) = { **a** }

FIRST (**c** *S*) = { **c** }

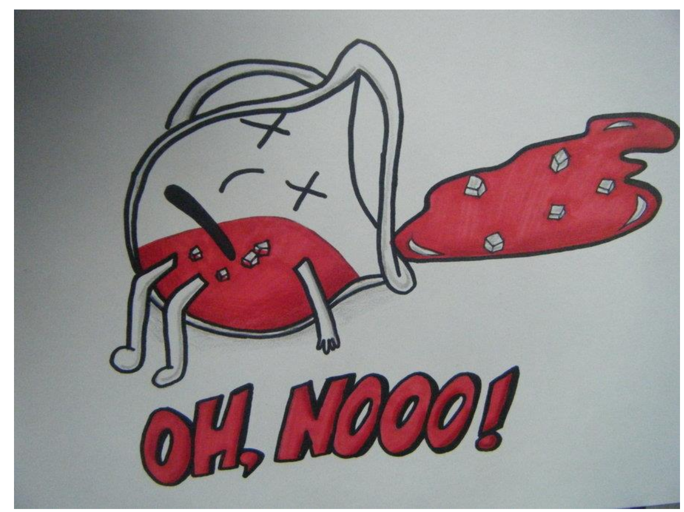
FOLLOW (S) = { **eof, c** }

FOLLOW (B) = { **c, eof** }

FOLLOW (D) = { **a, c** }

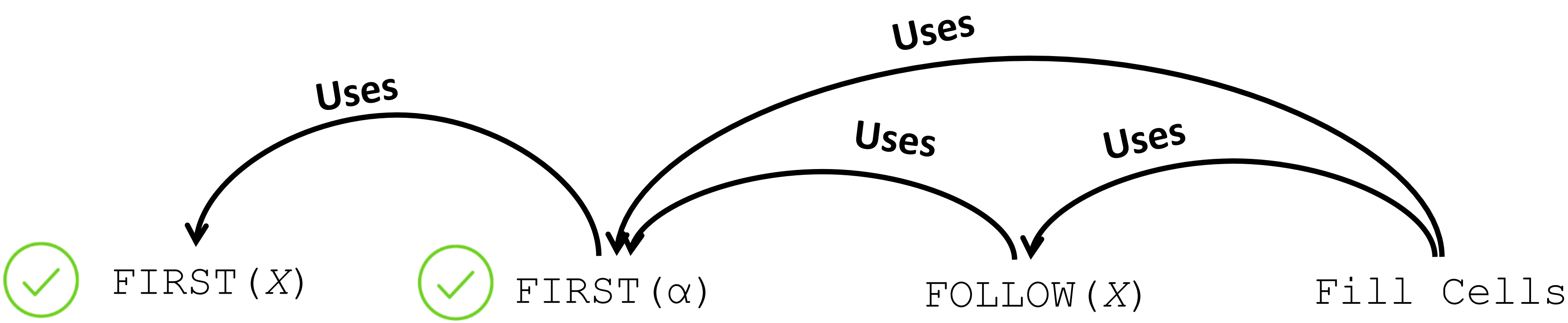
	a	**b**	**c**	**d**	**eof**
S	*D B* *B* **c**		*D B* *B* **c**		
B	**a b**				
D	*ε*		*ε*		

Another Collision! Grammar is still not LL(1)



Review: Selector Table Dependencies

Review Lecture 9 – FIRST Sets



Step 1.
Build FIRST sets for single symbols of the grammar

Step 2.
Build FIRST sets for RHS symbols strings

Step 3.
Build FOLLOW sets for Nonterminals

Step 4.
Place productions

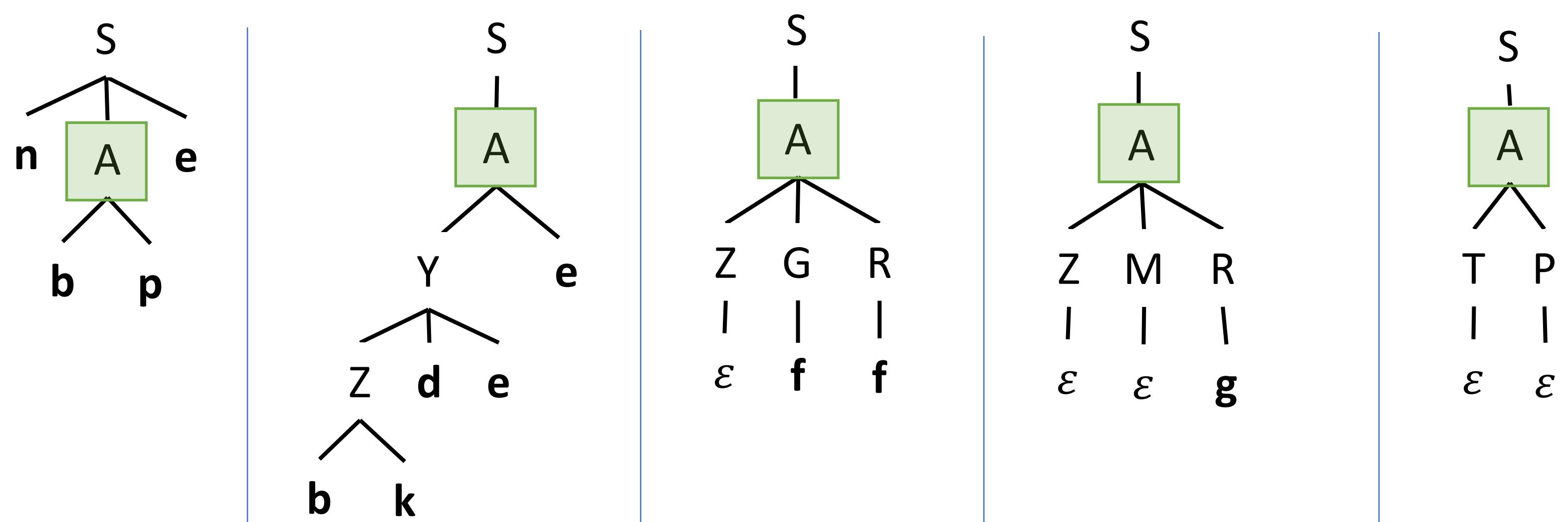
Last Time

Today

A Parse Tree Perspective

Building LL(1) Selector Table: FIRST sets, single symbol

FIRST(X): The set of terminals that begin strings derivable from X, and also, if X can derive ε, then ε is in FIRST(X).



b ∈ FIRST(A) | **b** ∈ FIRST(A) | **f** ∈ FIRST(A) | **g** ∈ FIRST(A) | ε ∈ FIRST(A)