

# Check-in

Review - ASTs

**Show the AST for the expression**

$$(1 + x) * 2$$

# Administrivia

Unflipped Wednesday

University of Kansas | Drew Davidson

*ECS 665*

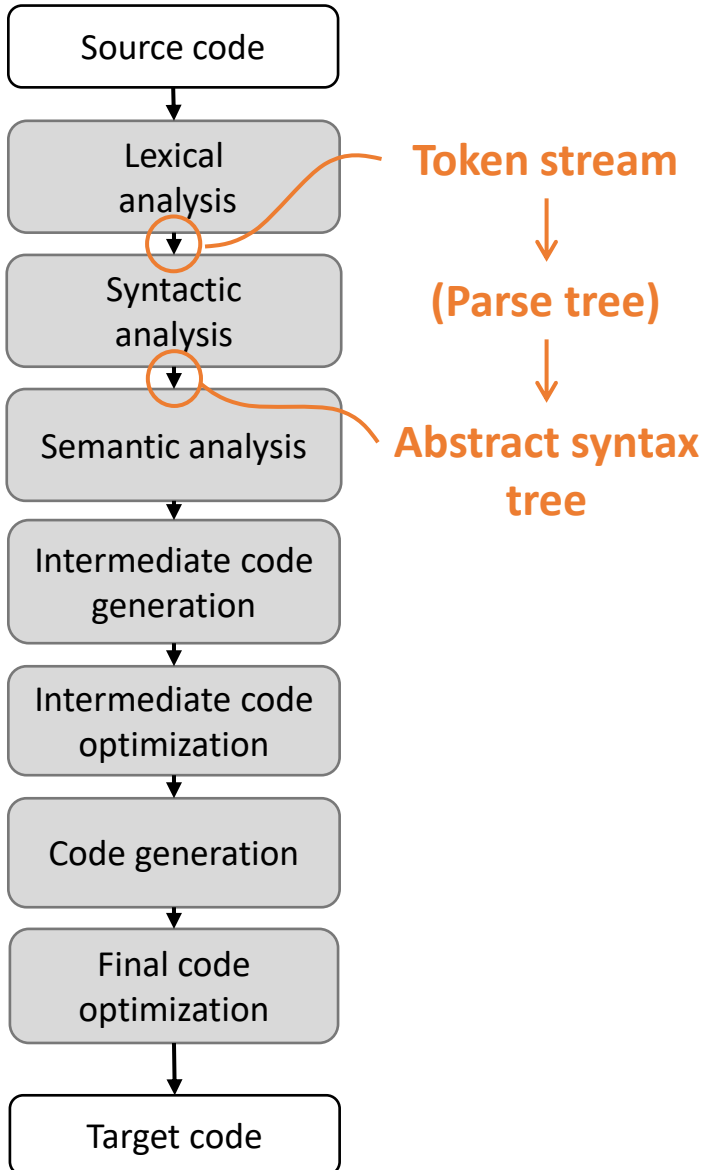
**COMPILER**

***CONSTRUCTION***

(Predictive) Parsing

# Compiler Construction

## Progress Pics



## Syntactic Analysis

- We know *what* we want (induce an AST from a given token stream)

# Last Time

Review - ASTs

## Syntax-Directed Definition

- Showed how to use SDD to induce program's AST
- We can *specify* a correct syntactic analysis

### You Should Know

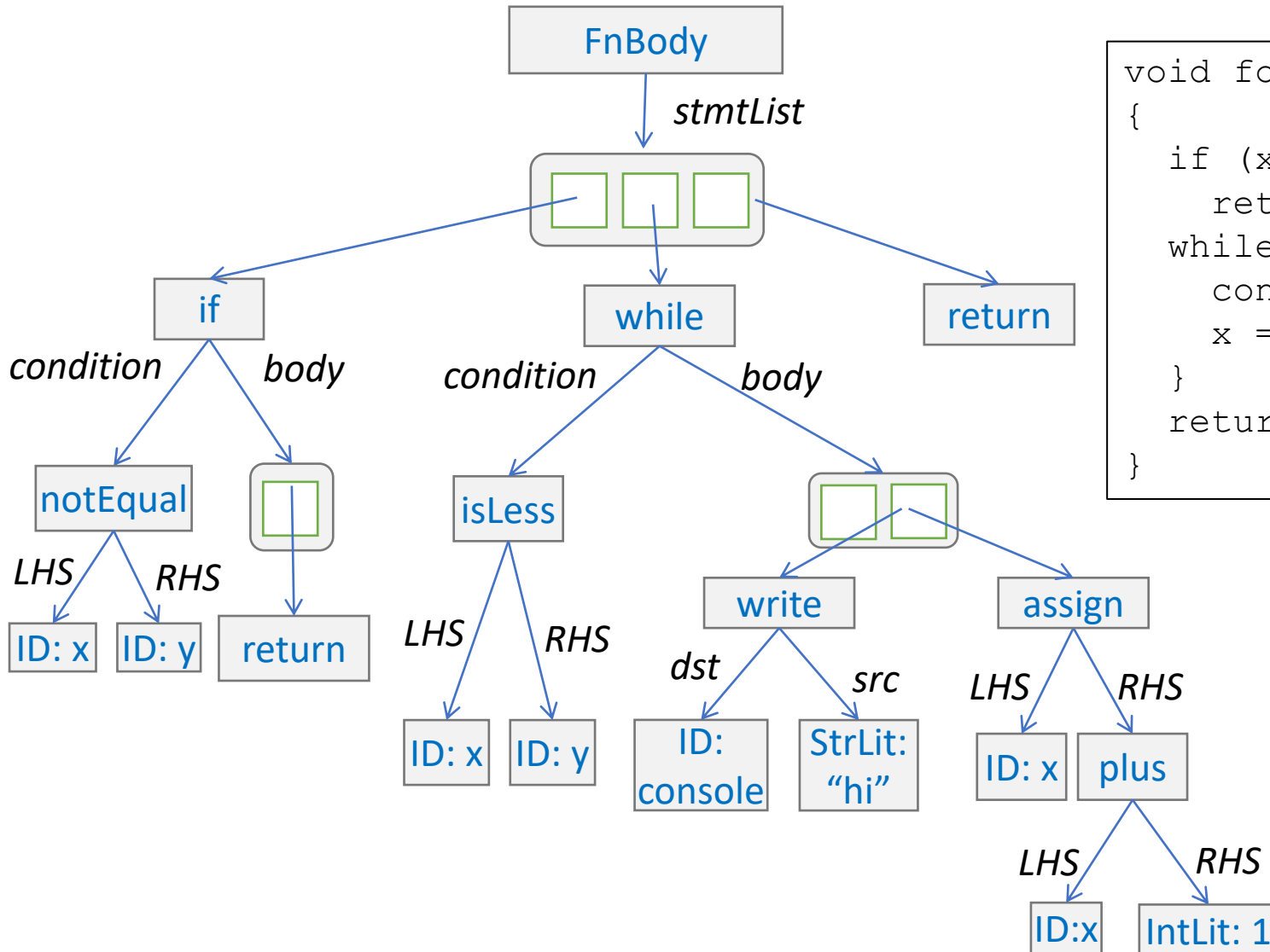
- The concepts of Parse Trees / ASTs
- How to specify syntax-directed definitions (to induce scalar values, data structures, etc. from parse trees)



**Syntax-Directed  
Definition**

# AST For Input Code Snippet

Abstract Syntax Trees – ASTs in Code



```
void foo(int x, int y)
{
    if (x != y)
        return;
    while ( x < y){
        console << "hi";
        x = (x + 1);
    }
    return;
}
```

# The Goal and The Path

Abstract Syntax Trees – ASTs in Code

**Goal:** ASTs for input programs

**Specification:** Syntax-directed Definitions

**Implementation:** ???

**Us**

# COMPILER

# LAND

Code Generation

Execution

Runtime Environment

Intermediate Representation

Optimization

SDD

Semantics

Parsing

Lexical Analysis

Syntactic Definiton





# Today's Outline

(Predictive) Parsing

## Parsing

- Complexity

## The LL(k) Languages

- Intro
- LL(1) parsing
- LL(1) transformations



Parsing

# The Price of Expressive Power

(Predictive) Parsing – Parsing Complexity

## Parsing is costly

- Jumping from Regular Languages to CF-Languages incurs penalties
  - Lose lots of language properties
  - Complexity for some operations goes up



*Power always comes at a price*

# Parsing: Bad News/Good News

(Predictive) Parsing – Parsing Complexity

## Bad News

- Complexity - CFL recognition:
  - CYK:  $O(n^3)$
  - Best known bound:  $O(n^{2.3728639})$
- Parsing is at least as hard

## Good News

- We don't need to recognize all CFLs
  - We can assume ambiguity-free syntax
  - Exchange expressiveness for faster parsers



# Scaling Back Expressive Power

(Predictive) Parsing – Parsing Complexity

## Compilers aren't concerned with arbitrary languages

- Programming languages *should* be unambiguous
- Complex operations can be built from simple syntax

*Paying for features we don't need!*



“Don't buy the truck if you ain't haulin' freight”  
-Drew Davidson

# Restricting the Language Class

*(Predictive) Parsing – Parsing Complexity*

- Allows us to...
  - Build linear-time parsers
  - Detect ambiguity
- Let's consider one such class of restrictions



***Disallowing Heavyweight Languages***

# Today's Outline

*Preview Lecture 8 – Predictive Parsing*

## Parsing

- Complexity

## The LL(k) languages

- Intro
- LL(1) parsing
- LL(1) transformations

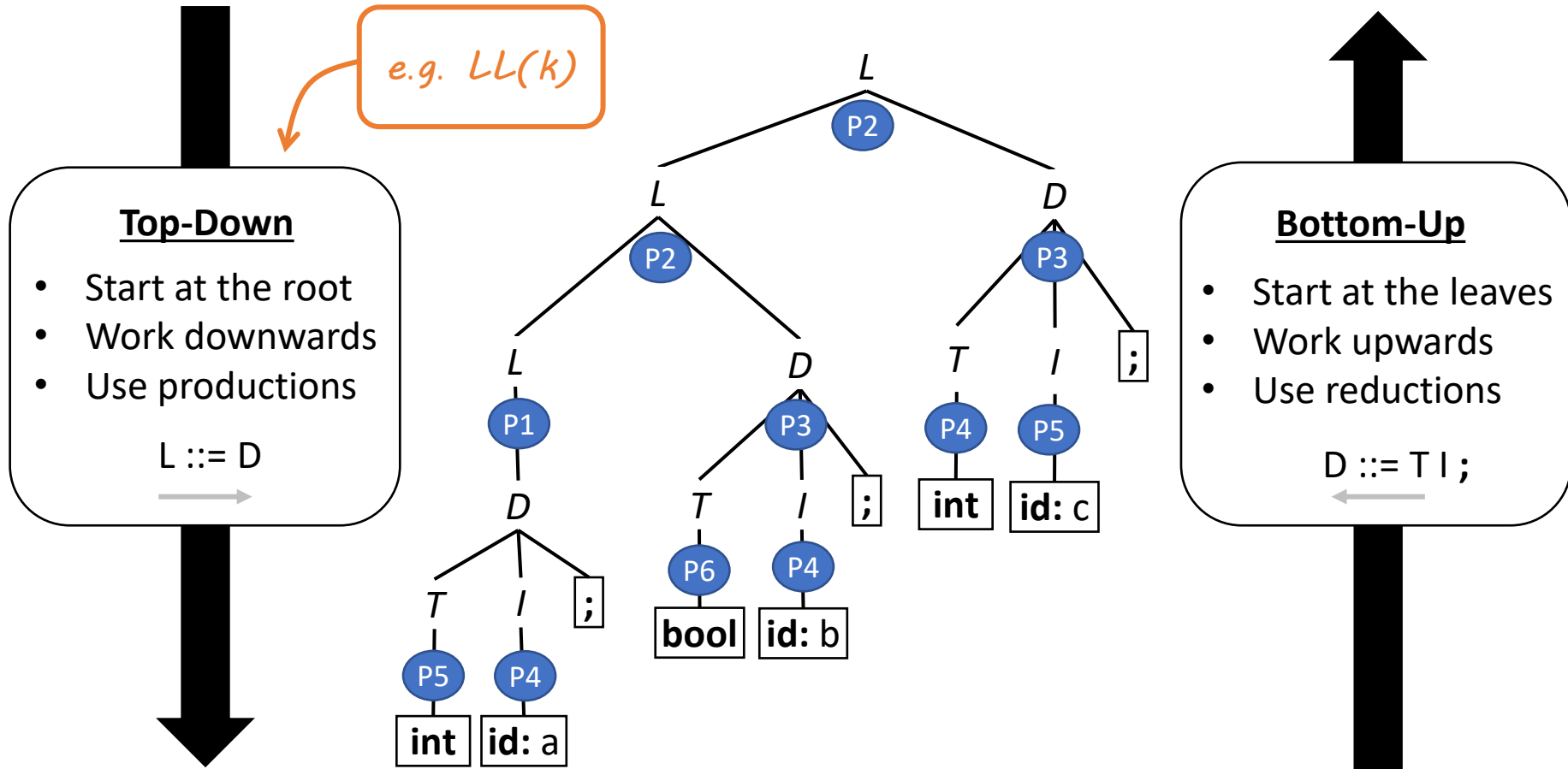


***Toyota Yaris: A sensible compact  
LL(1) languages are the Yaris of the  
Parsing world***

# Parser Type

## LL(k) Parsers - Definitions

### Most parsers classified by how they build the parse tree



# Predictive Parsers

## *LL(k) Parsers - Definitions*

- “Predict” correct production for the given nonterminal before seeing all RHS symbols
- Based on input prefix  
“Guess and check” the suffix





# LL Grammars: Anatomy of Notation

## LL Grammars - Intro

*2<sup>nd</sup> L: Performs  
Leftmost derivation*

LL (k)

*Lookahead:  
Number of tokens  
needed to choose  
next production*

*1<sup>st</sup> L: Single  
Left-to-right Scan  
of token stream*

Bigger K means more  
expressive languages

# LL Grammar Examples

## LL Grammars - Intro

### LL(0) Grammars

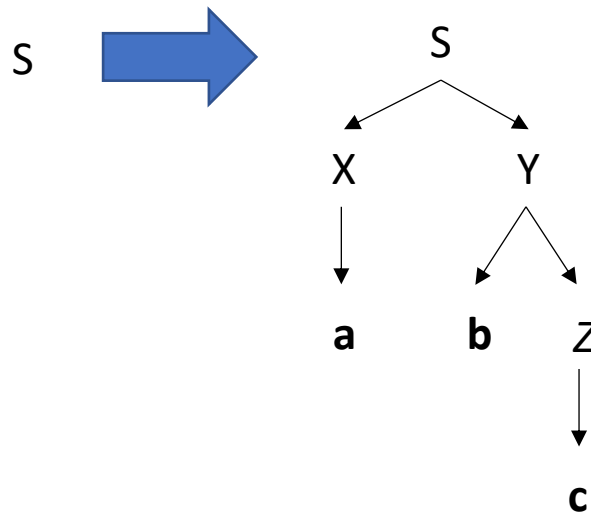
Takes leftmost derivation  
step knowing **zero**  
underived input tokens

$S ::= X Y$

$X ::= a$

$Y ::= b Z$

$Z ::= c$



# LL Grammar Examples

## LL Grammars - Intro

### LL(0) Grammars

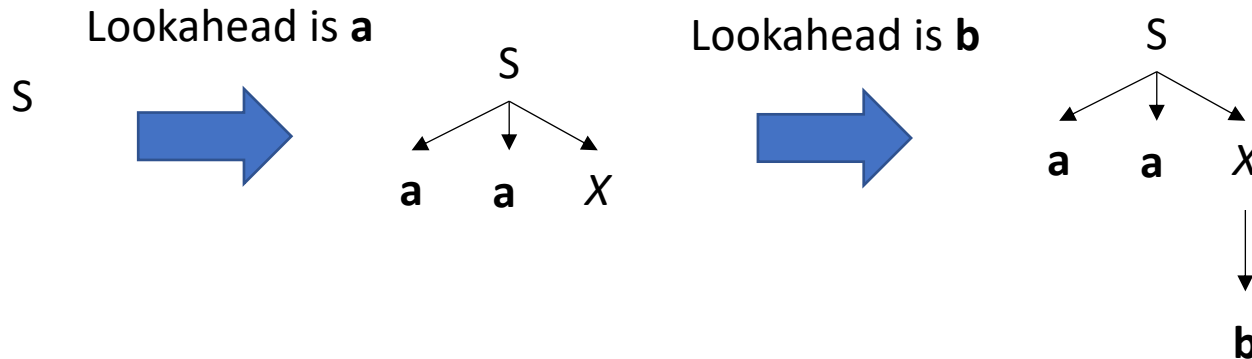
Takes leftmost derivation  
step knowing **zero**  
underived input tokens

$S ::= XY$   
 $X ::= a$   
 $Y ::= bZ$   
 $Z ::= c$

### LL(1) Grammars

Takes leftmost derivation  
step knowing **one**  
underived input token

$S ::= a a X$   
          |  $b X$   
 $X ::= a$   
          |  $b$



# LL Grammar Examples

## LL Grammars - Intro

### LL(0) Grammars

Takes leftmost derivation  
step knowing **zero**  
underived input tokens

$$\begin{aligned} S &::= X Y \\ X &::= a \\ Y &::= b Z \\ Z &::= c \end{aligned}$$

### LL(1) Grammars

Takes leftmost derivation  
step knowing **one**  
underived input token

$$\begin{aligned} S &::= a a X \\ &\quad | b X \\ X &::= a \\ &\quad | b \end{aligned}$$

### LL(2) Grammars

Takes leftmost derivation  
step knowing **two**  
underived input tokens

$$\begin{aligned} S &::= a a \\ &\quad | a b \end{aligned}$$

(Lookahead **a** is insufficient)

Lookahead is **a a**



# LL Grammar Examples

## *LL Grammars - Intro*

### LL(1) Grammars



Takes leftmost derivation  
step knowing **one**  
underived input token

Row:  
leftmost leaf  
nonterminal


Column  
Lookahead  
token

Cell:  
Production

# LL Grammar Examples

## LL Grammars - Intro

With some grammars you can just eyeball the selector table

$$S ::= (S) \\ | \{S\} \\ | \epsilon$$


Column  
Lookahead  
token

Row:  
leftmost leaf  
nonterminal

	(	{	)	}
S	(S)	{S}	$\epsilon$	$\epsilon$

Cell:  
Production

# Today's Outline

## *(Predictive) Parsing*

### **Parsing**

- Complexity

### **A New Type of Language - LL**

- Intro
- LL(1) parsing
- LL(1) transformations



**Parsing**

# LL(1) Algorithm Psuedocode

## Predictive Parsing - LL(1) Parsing

```
stack.push(eof)  
stack.push(Start non-term)  
lookahead = scanner.first_token()
```

} *Initialization*

Repeat

```
  if stack.top is a terminal  
    match stack.top with lookahead  
    pop y from the stack  
    lookahead = scanner.next_token()
```

} *Case 1  
Terminal on stack:  
check prediction*

```
  if stack.top is a nonterminal  
    X = stack.pop()  
    get P = table[X,lookahead]  
    push P's RHS symbols Right-to-Left
```

} *Case 2  
Non-terminal on stack:  
Derive new prediction*

Until one of the following:

```
  stack is empty (accept)  
  stack.top is a terminal that doesn't match t (reject)  
  stack.top is a non-term and table entry is empty (reject)
```

} *Exit*



## Example Grammar

$S ::= ( S )$   
 $\quad | \{ S \}$   
 $\quad | \epsilon$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

### Initialization

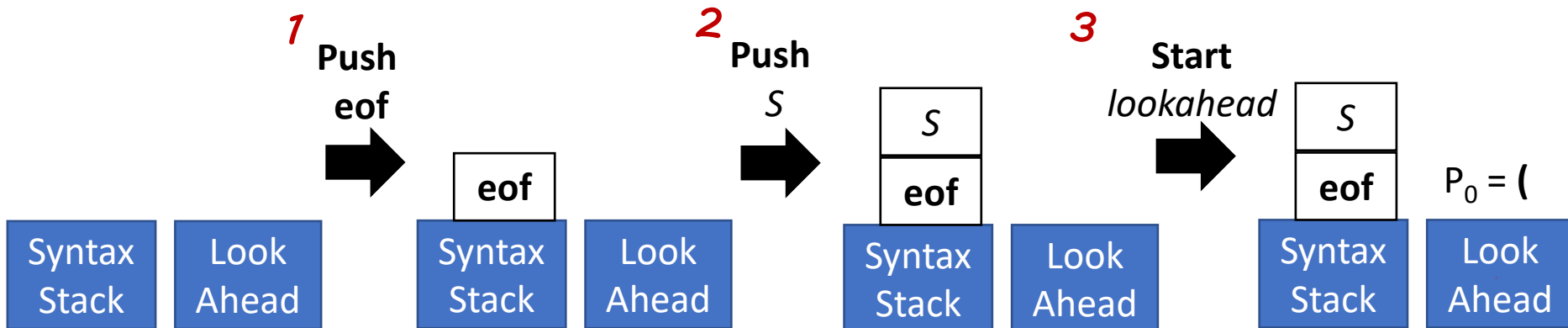
- 1 `stack.push(eof)`
- 2 `stack.push(Start non-term)`
- 3 `lookahead = scanner.first_token()`

### Selector Table

	(	)	{	}
S	(S)	$\epsilon$	{S}	$\epsilon$

### Input Stream

( { } ) **eof**  
P<sub>0</sub> P<sub>1</sub> P<sub>2</sub> P<sub>3</sub> P<sub>4</sub>



## Example Grammar

$S ::= ( S )$   
 $| \{ S \}$   
 $| \epsilon$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

### Initialization

- 1 `stack.push(eof)`
- 2 `stack.push(Start non-term)`
- 3 `lookahead = scanner.first_token()` “Predicted”

### Selector Table

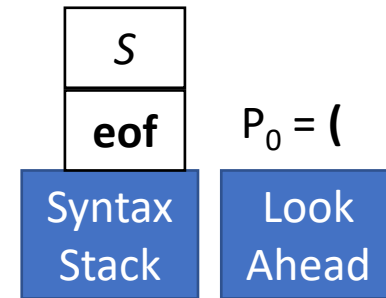
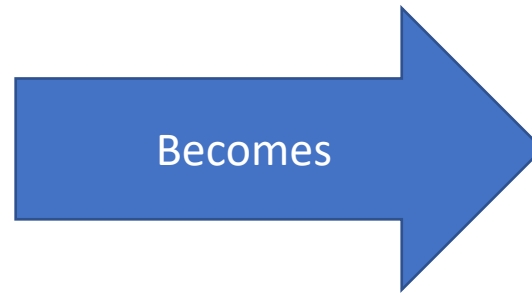
	(	)	{	}
S	(S)	$\epsilon$	{S}	$\epsilon$

### Input Stream

( { } ) **eof**  
P<sub>0</sub> P<sub>1</sub> P<sub>2</sub> P<sub>3</sub> P<sub>4</sub>

### Derivation Sequence

S



## Example Grammar

$$S ::= ( S )$$

$$| \{ S \}$$

$$| \epsilon$$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Nonterminal on stack:

- 1  $X = \text{pop}(\text{stack})$
- 2 get  $P = \text{table}[X, \text{lookahead}]$
- 3 push  $P$ 's symbols *Right-to-Left*

### Selector Table

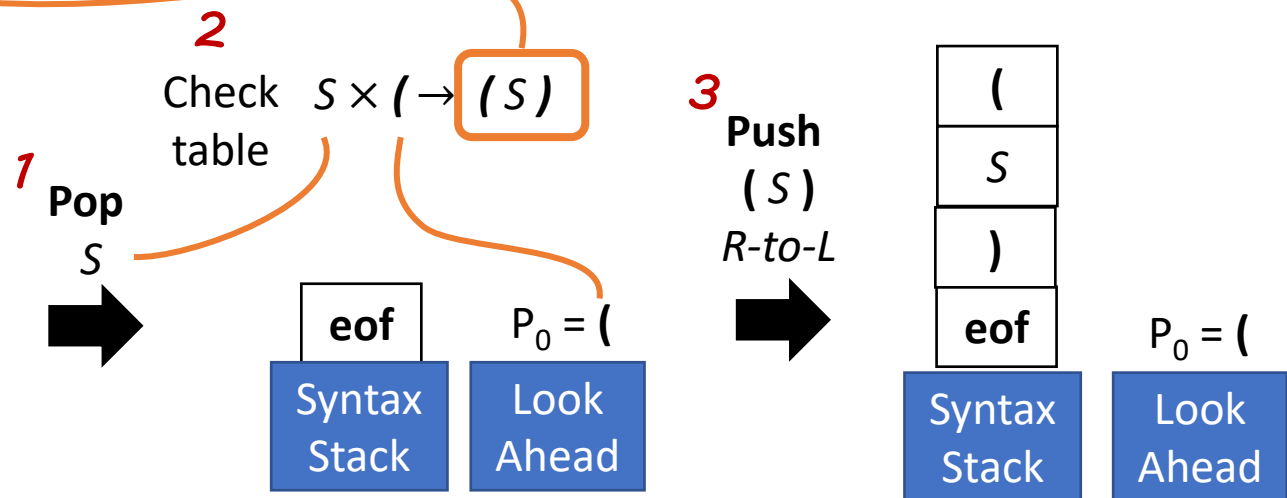
	(	)	{	}
S	(S)	$\epsilon$	{S}	$\epsilon$

### Input Stream

(	{	}	)	eof
$P_0$	$P_1$	$P_2$	$P_3$	$P_4$

### "Predicted" Derivation Sequence

S



## Example Grammar

$S ::= ( S )$   
 $| \{ S \}$   
 $| \epsilon$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing  
*Nonterminal on stack:*

- 1  $X = \text{pop}(\text{stack})$
- 2  $\text{get } P = \text{table}[X, \text{lookahead}]$
- 3  $\text{push } P\text{'s symbols } \textit{Right-to-Left}$

### Selector Table

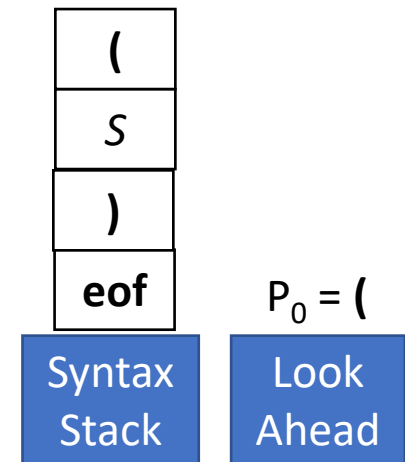
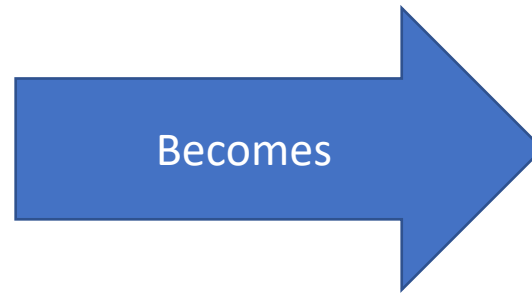
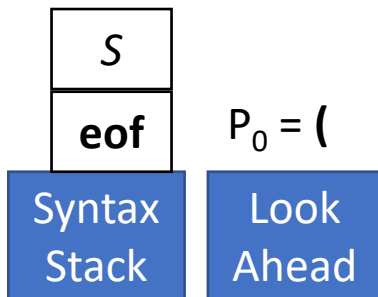
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

### Input Stream

( { } ) eof  
 $P_0$   $P_1$   $P_2$   $P_3$   $P_4$

### “Predicted” Derivation Sequence

$S \Rightarrow ( S )$



## Example Grammar

$$S ::= ( S )$$

$$| \{ S \}$$

$$| \epsilon$$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next\_token() “Predicted”

### Selector Table

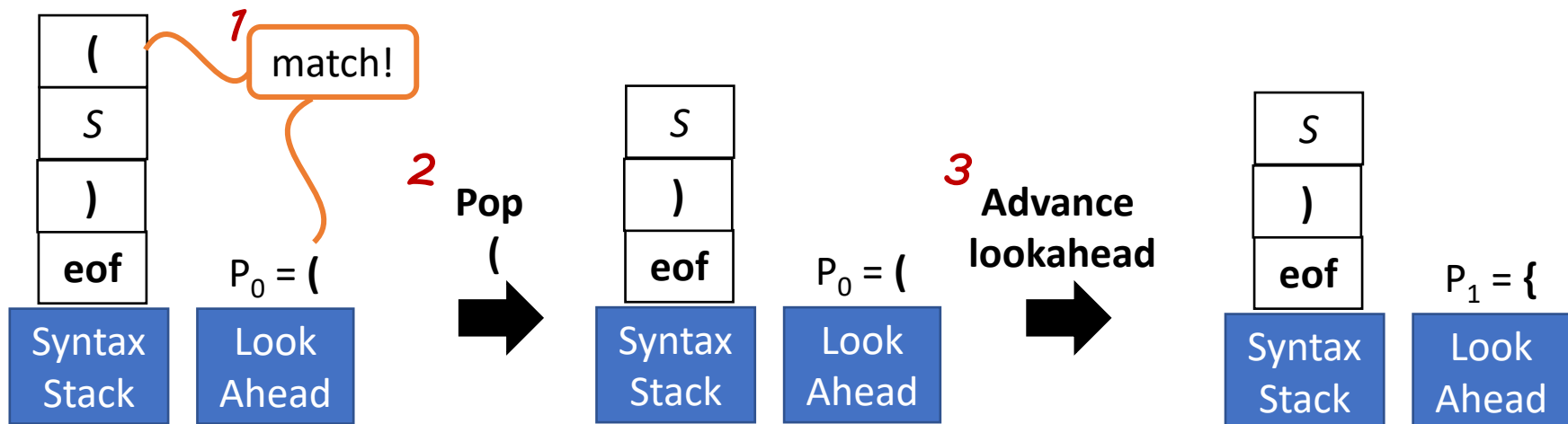
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

### Input Stream

(	{	}	)	eof
P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>

### Derivation Sequence

S ⇒ ( S )



## Example Grammar

$S ::= ( S )$   
 $\quad | \{ S \}$   
 $\quad | \epsilon$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

*Terminal on stack:*

- 1 match `stack.top` with lookahead
- 2 pop stack
- 3 lookahead = `scanner.next_token()` “Predicted”

## Selector Table

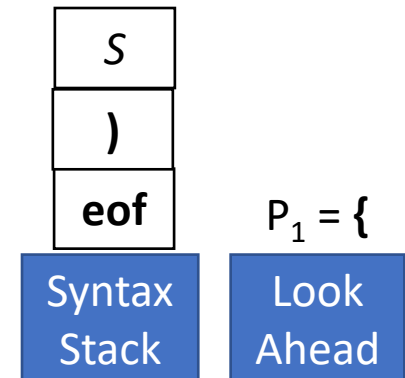
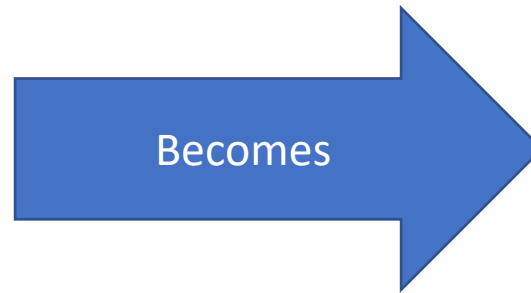
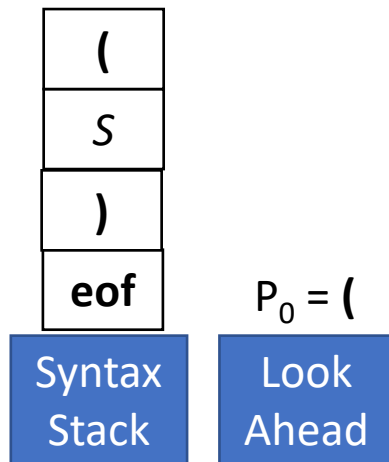
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

## Input Stream

( { } ) eof  
P<sub>0</sub> P<sub>1</sub> P<sub>2</sub> P<sub>3</sub> P<sub>4</sub>  
✓

## Derivation Sequence

$S \Rightarrow ( S )$



## Example Grammar

$$S ::= ( S )$$

$$| \{ S \}$$

$$| \epsilon$$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Nonterminal on stack:

- 1  $X = \text{pop}(\text{stack})$
- 2 get  $P = \text{table}[X, \text{lookahead}]$
- 3 push  $P$ 's RHS symbols *Right-to-Left*

### Selector Table

	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

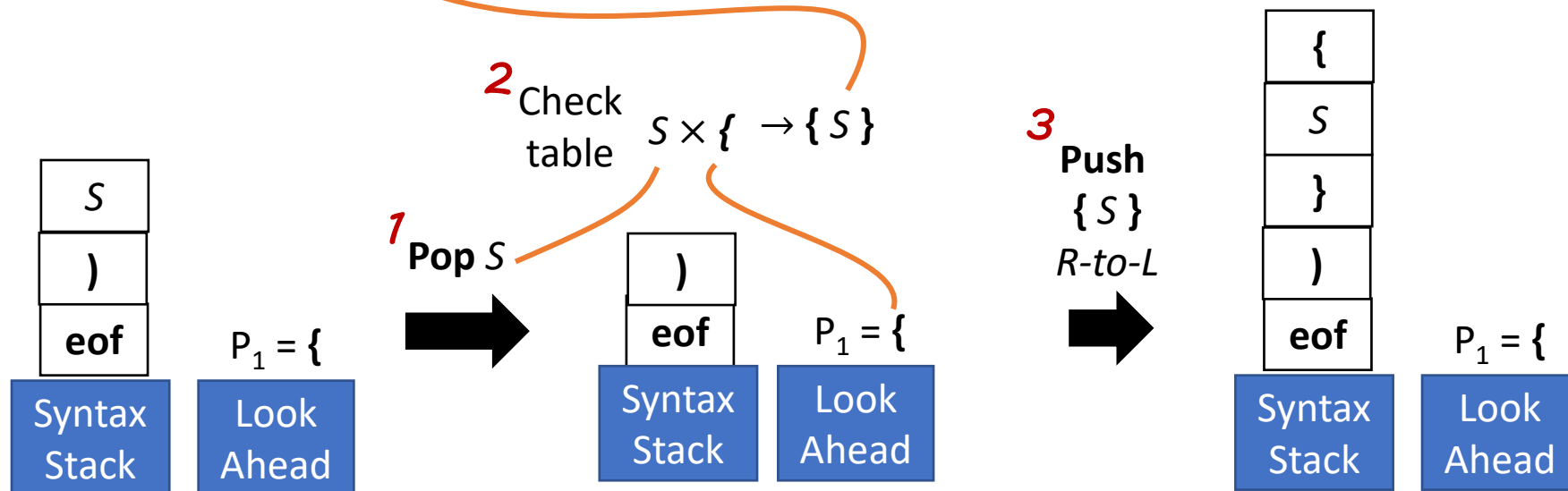
### Input Stream

(	{	}	)	eof
$P_0$	$P_1$	$P_2$	$P_3$	$P_4$

✓

### "Predicted" Derivation Sequence

$S \Rightarrow ( S )$



## Example Grammar

$$S ::= ( S )$$
$$| \{ S \}$$
$$| \epsilon$$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Nonterminal on stack:

- 1  $X = \text{pop}(\text{stack})$
- 2 get  $P = \text{table}[X, \text{lookahead}]$
- 3 push  $P$ 's RHS symbols *Right-to-Left*

## Selector Table

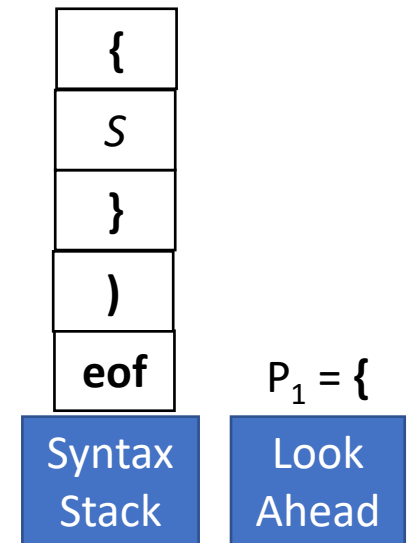
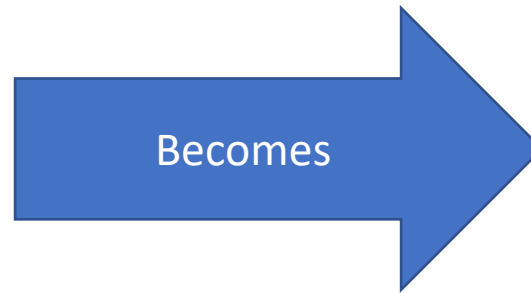
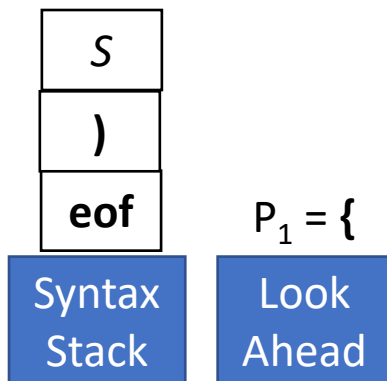
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

## Input Stream

( { } ) eof  
 $P_0$   $P_1$   $P_2$   $P_3$   $P_4$   
✓

## "Predicted" Derivation Sequence

$S \Rightarrow ( S )$   
 $\Rightarrow ( \{ S \} )$





## Example Grammar

$$S ::= ( S )$$
$$| \{ S \}$$
$$| \epsilon$$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next\_token()

## Selector Table

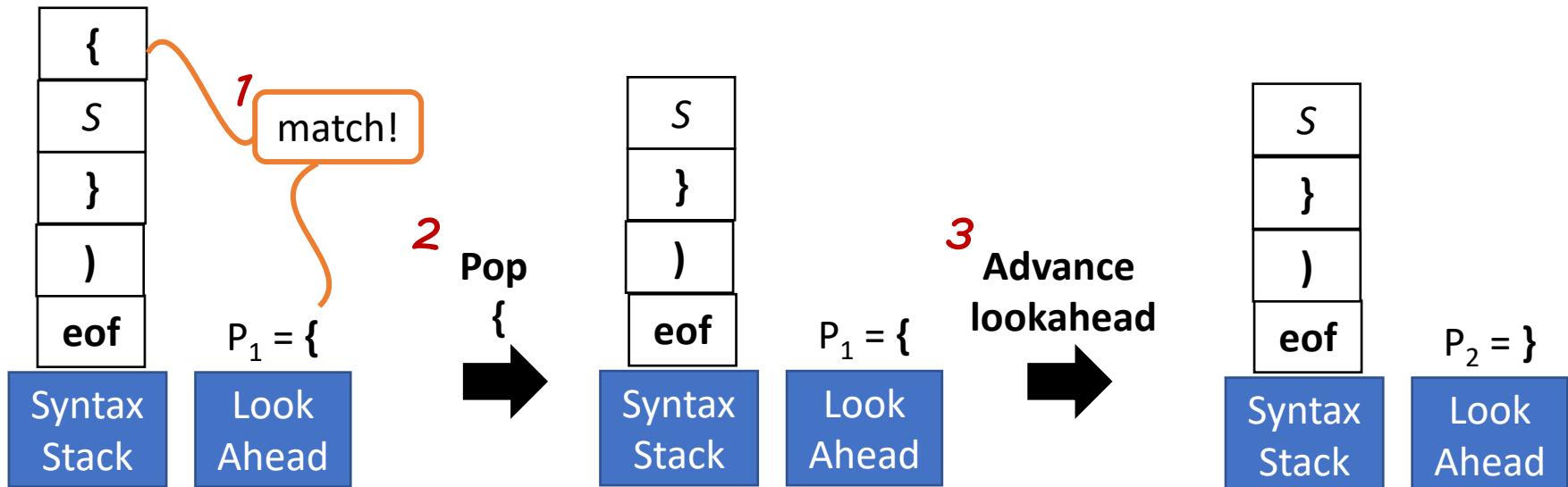
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

## Input Stream

( { } ) eof  
P<sub>0</sub> P<sub>1</sub> P<sub>2</sub> P<sub>3</sub> P<sub>4</sub>  
✓

## “Predicted” Derivation Sequence

S  $\Rightarrow$  ( S )  
 $\Rightarrow$  ( { S } )



## Example Grammar

$$S ::= ( S )$$
$$| \{ S \}$$
$$| \epsilon$$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsin

Terminal on stack:

- 1 match `stack.top` with lookahead
- 2 pop stack
- 3 lookahead = `scanner.next_token()`

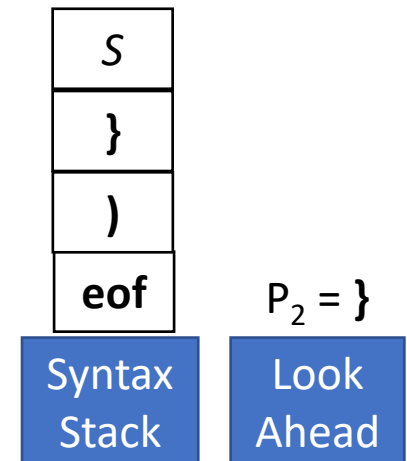
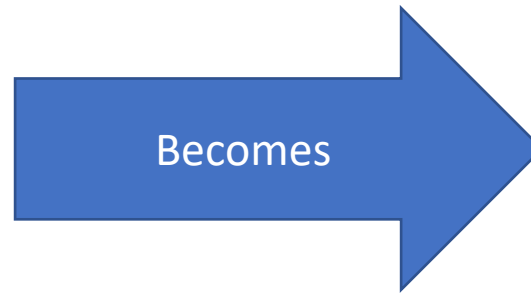
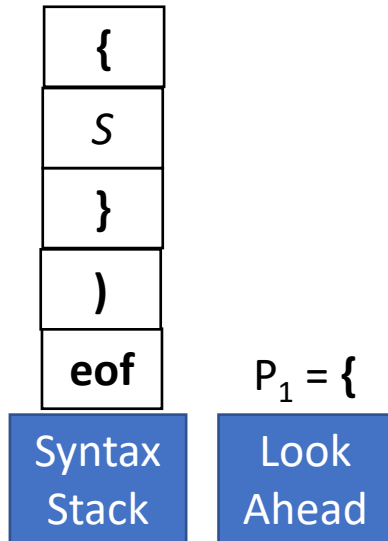
## Selector Table

	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

## Input Stream

(	{	}	)	eof
P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>
✓	✓			

## "Predicted" Derivation Sequence

$$S \Rightarrow ( S )$$
$$\Rightarrow ( \{ S \} )$$


## Example Grammar

$$S ::= ( S )$$

$$| \{ S \}$$

$$| \epsilon$$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsin

Nonterminal on stack:

- 1  $X = \text{pop}(\text{stack})$
- 2 get  $P = \text{table}[X, \text{lookahead}]$
- 3 push  $P$ 's RHS symbols *Right-to-Left* "Predicted"

### Selector Table

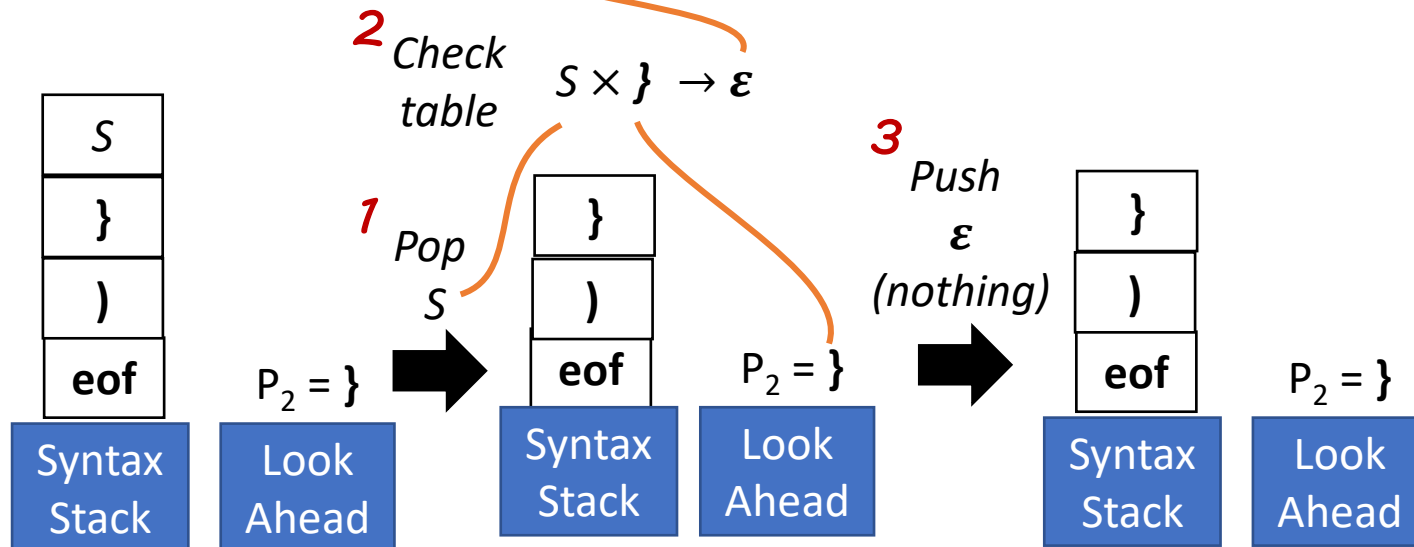
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

### Input Stream

(	{	}	)	eof
$P_0$	$P_1$	$P_2$	$P_3$	$P_4$
✓	✓			

### Derivation Sequence

$$S \Rightarrow ( S )$$

$$\Rightarrow ( \{ S \} )$$


## Example Grammar

$$S ::= ( S )$$

$$| \{ S \}$$

$$| \epsilon$$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Nonterminal on stack:

- 1  $X = \text{pop}(\text{stack})$
- 2 get  $P = \text{table}[X, \text{lookahead}]$
- 3 push  $P$ 's RHS symbols *Right-to-Left* "Predicted"

### Selector Table

	(	)	{	}
S	(S)	$\epsilon$	{S}	$\epsilon$

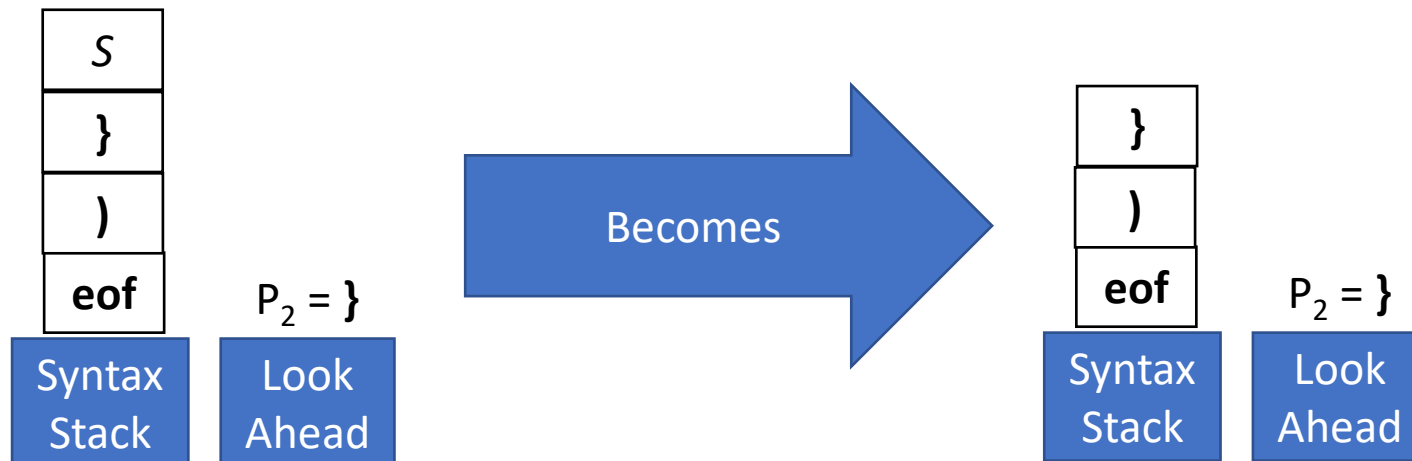
### Input Stream

(	{	}	)	eof
$P_0$	$P_1$	$P_2$	$P_3$	$P_4$
✓	✓			

### Derivation Sequence

$$S \Rightarrow ( S )$$

$$\Rightarrow ( \{ S \} )$$

$$\Rightarrow ( \{ \} )$$


## Example Grammar

$S ::= ( S )$   
|  $\{ S \}$   
|  $\epsilon$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next\_token()

## Selector Table

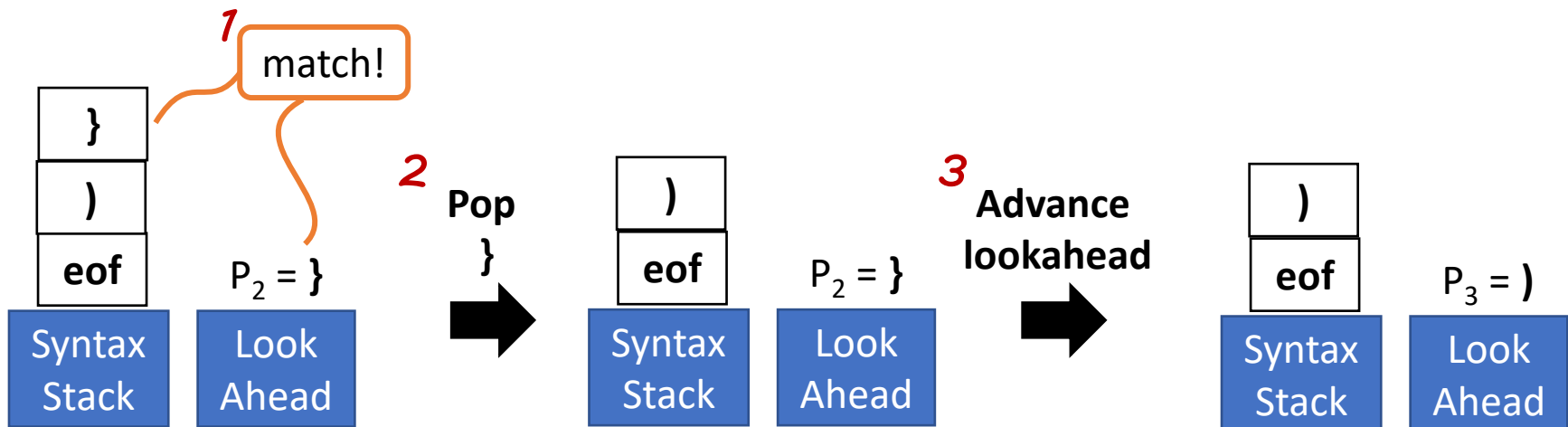
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

## Input Stream

(	{	}	)	eof
P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>
✓	✓	✓		

## "Predicted" Derivation Sequence

$S \Rightarrow ( S )$   
 $\Rightarrow ( \{ S \} )$   
 $\Rightarrow ( \{ \} )$



## Example Grammar

$S ::= ( S )$   
|  $\{ S \}$   
|  $\epsilon$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

*Terminal on stack:*

- 1 match `stack.top` with lookahead
- 2 pop stack
- 3 lookahead = `scanner.next_token()`

## Selector Table

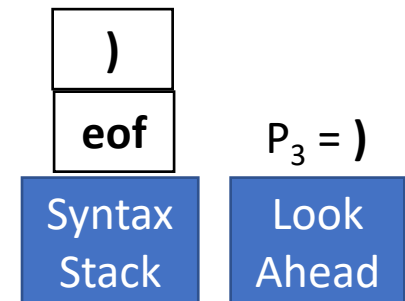
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

## Input Stream

( { } ) eof  
P<sub>0</sub> P<sub>1</sub> P<sub>2</sub> P<sub>3</sub> P<sub>4</sub>  
✓ ✓ ✓

## “Predicted” Derivation Sequence

$S \Rightarrow ( S )$   
 $\Rightarrow ( \{ S \} )$   
 $\Rightarrow ( \{ \} )$



## Example Grammar

$S ::= ( S )$   
 $| \{ S \}$   
 $| \epsilon$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next\_token()

## Selector Table

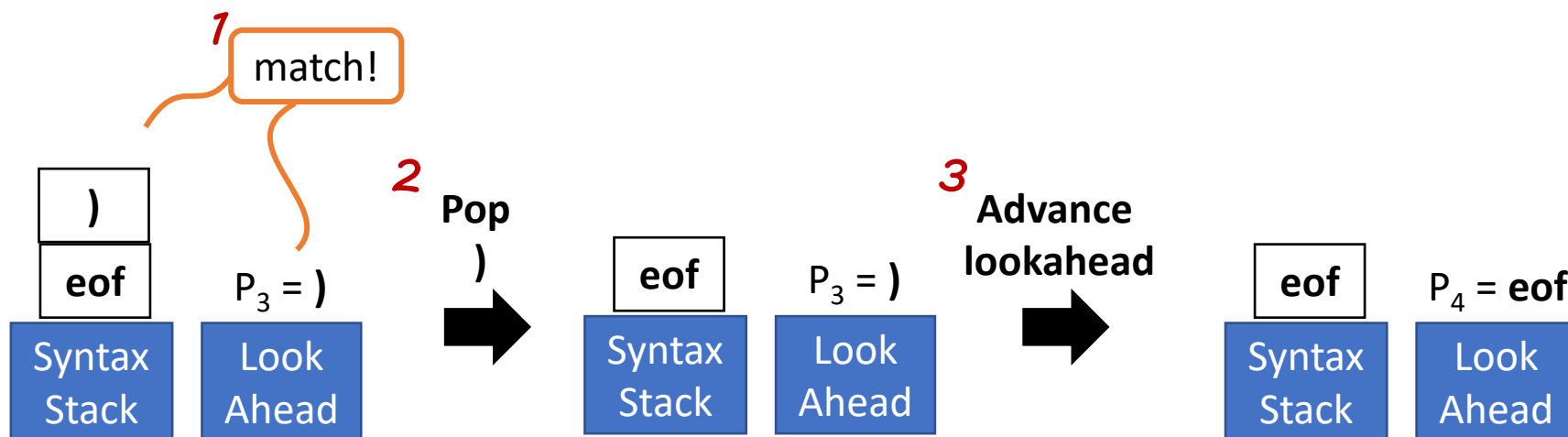
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

## Input Stream

(	{	}	)	eof
P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>
✓	✓	✓		

## "Predicted" Derivation Sequence

$S \Rightarrow ( S )$   
 $\Rightarrow ( \{ S \} )$   
 $\Rightarrow ( \{ \} )$



## Example Grammar

$S ::= ( S )$   
|  $\{ S \}$   
|  $\epsilon$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

*Terminal on stack:*

- 1 match `stack.top` with lookahead
- 2 pop stack
- 3 lookahead = `scanner.next_token()`

## Selector Table

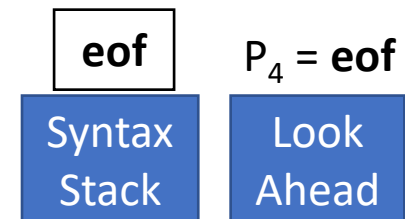
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

## Input Stream

( { } ) eof  
P<sub>0</sub> P<sub>1</sub> P<sub>2</sub> P<sub>3</sub> P<sub>4</sub>  
✓ ✓ ✓ ✓

## “Predicted” Derivation Sequence

$S \Rightarrow ( S )$   
 $\Rightarrow ( \{ S \} )$   
 $\Rightarrow ( \{ \} )$





## Example Grammar

$S ::= ( S )$   
 $| \{ S \}$   
 $| \epsilon$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match `stack.top` with lookahead
- 2 pop stack
- 3 lookahead = `scanner.next_token()`

## Selector Table

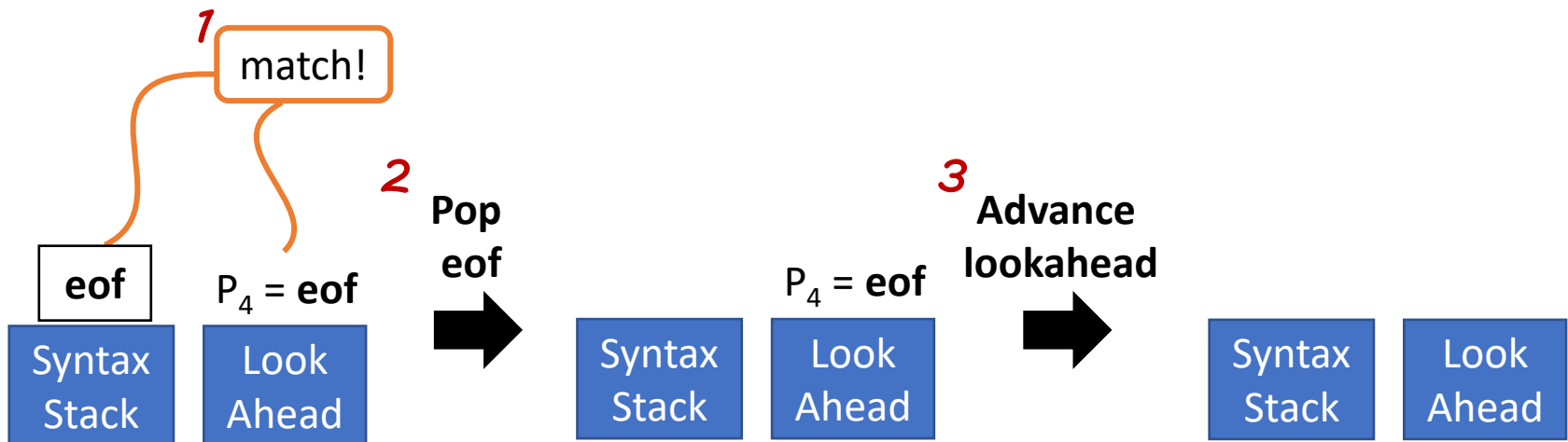
	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

## Input Stream

(	{	}	)	eof
P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>
✓	✓	✓	✓	✓

## "Predicted" Derivation Sequence

$S \Rightarrow ( S )$   
 $\Rightarrow ( \{ S \} )$   
 $\Rightarrow ( \{ \} )$



## Example Grammar

$S ::= ( S )$   
|  $\{ S \}$   
|  $\epsilon$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

*Terminal on stack:*

- 1 match `stack.top` with lookahead
- 2 pop stack
- 3 lookahead = `scanner.next_token()`

## Selector Table

	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

## Input Stream

(	{	}	)	eof
P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>
✓	✓	✓	✓	✓

## “Predicted” Derivation Sequence

$S \Rightarrow ( S )$   
 $\Rightarrow ( \{ S \} )$   
 $\Rightarrow ( \{ \} )$

Syntax  
Stack

Look  
Ahead

## Example Grammar

$$S ::= ( S )$$
$$| \{ S \}$$
$$| \epsilon$$

# LL(1) Example

(Predictive) Parsing - LL(1) Parsing

*Exit*

Stack and Lookahead both empty:  
Accept!

### Selector Table

	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

### Input Stream

(	{	}	)	eof
P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>
=({)	=({)	=({)	=({)	=({)

### "Predicted" Derivation Sequence

$$S \Rightarrow ( S )$$
$$\Rightarrow ( \{ S \} )$$
$$\Rightarrow ( \{ \} )$$


Syntax  
Stack

Look  
Ahead

# Running LL(1)

*(Predictive) Parsing - LL(1) Parsing*

## **Linear parse!**

- Once we verify a position we never look at it again

## **Pretty fast given general parsing complexity**

- Especially once you start combining steps

## **Let's do another quick example**

- This time we'll show rejecting bad input

## Example Grammar

$$S ::= ( S )$$
$$| \{ S \}$$
$$| \epsilon$$

# LL(1) Reject Example

(Predictive) Parsing - LL(1) Parsing

### Selector Table

	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

### Input Stream

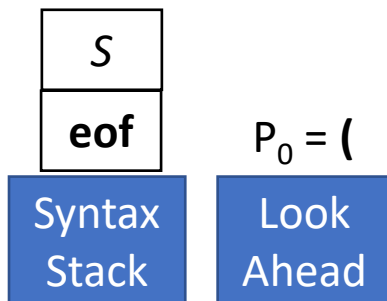
(	}	eof
P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>

### “Predicted” Derivation Sequence

$$S \Rightarrow ( S )$$

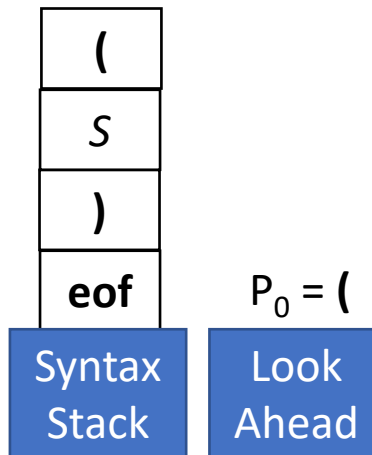
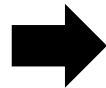
### Initialization

1,2,3



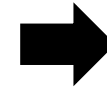
### Nonterminal Actions

1,2,3



### Terminal Actions

1,2,3



## Example Grammar

$$S ::= ( S )$$

$$| \{ S \}$$

$$| \epsilon$$

# LL(1) Reject Example

(Predictive) Parsing - LL(1) Parsing

### Selector Table

	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

### Input Stream

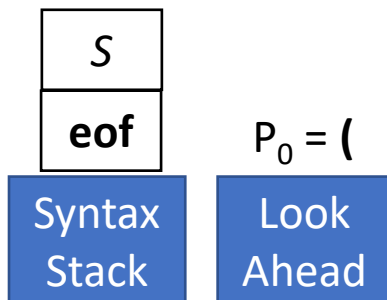
(	}	eof
P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>

### "Predicted" Derivation Sequence

$$S \Rightarrow ( S )$$

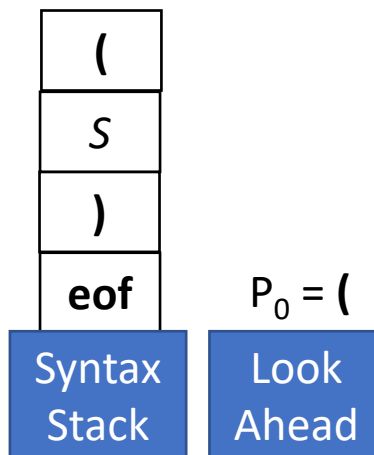
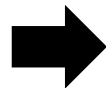
### Initialization

1,2,3



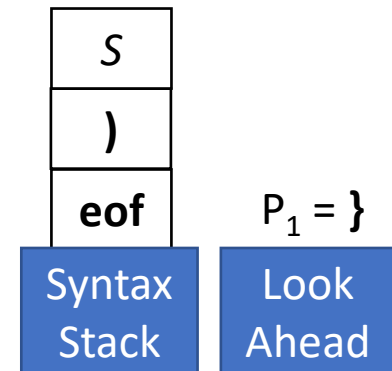
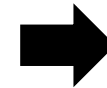
### Nonterminal Actions

1,2,3



### Terminal Actions

1,2,3



## Example Grammar

$$S ::= ( S )$$
$$| \{ S \}$$
$$| \epsilon$$

# LL(1) Reject Example

(Predictive) Parsing - LL(1) Parsing

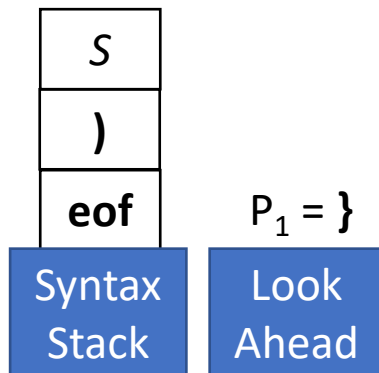
### Selector Table

	(	)	{	}
S	( S )	$\epsilon$	{ S }	$\epsilon$

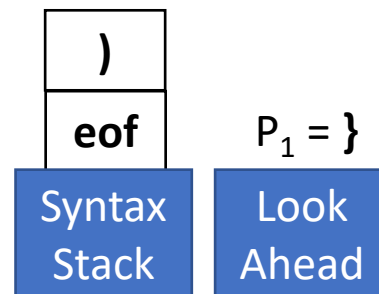
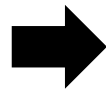
### Input Stream

(	}	eof
P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>

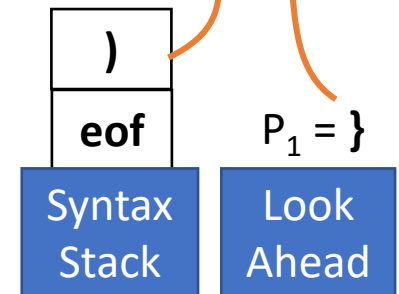
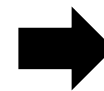
### "Predicted" Derivation Sequence

$$S \Rightarrow ( S )$$


Nonterminal  
Actions  
1,2,3



Terminal  
Actions  
1 - Fails!



# This Grammar is Great!

*(Predictive) Parsing - LL(1) Parsing*

With 1 token we know exactly what production it starts



...as long as the grammar is LL(1)



# Consider a Concerning Grammar...

*(Predictive) Parsing - LL(1) Parsing*

Our prior  
LL(2) Grammar

$S ::= a a$   
 $\quad | a b$



Equivalent  
LL(1) Grammar

$S ::= a X$   
 $X ::= a | b$

*The Language  
may be LL(1)  
Even when the  
Grammar is not LL(1)*

$S, a$

