

Check-In

Review: Abstract Interpretation

Draw the CFG for the following 3AC procedure. Indicate the IN and OUT sets for each basic block on a, b, c for a constant propagation analysis

Assume c is global and all other vars are local

```
fun_foo:    enter foo
           L1:    getarg 1 [a]
           L2:    [b] := 2
           L3:    [c] := 2
           L4:    [tmp0] := [a] LT64 3
           L5:    IFZ [tmp0] GOTO L11
           L6:    [tmp1] := [b] ADD64 7
           L7:    [b] := [tmp1]
           L8:    call bar
           L9:    [tmp3] := [c] ADD64 7
           L10:   [c] := [tmp3]
           L11:   setret [b]
           L12:   leave foo
```

Announcements

Administrivia

- Quiz 4 Friday
- Review Session Wednesday, 7:15 – 9:15 (I'll try to show up at 7:00)

Drew Davidson | University of Kansas

ECCS 665 **COMPILER** *CONSTRUCTION*

SSA

Previously...

Abstract Interpretation

Rounding out dataflow analysis concepts

- Some more examples
- Considering more complex code
- Dataflow Framework

Abstract Interpretation

- Concepts
- Examples

You should know

- The saturation approach to dataflow
- Handling loops, globals, large domains



Optimization

Today's Lecture Outline

SSA

Static Single Assignment

- Motivation
- Concept
- Importance
- Implementation



Optimization

Recall Data Allocation

SSA – Motivation

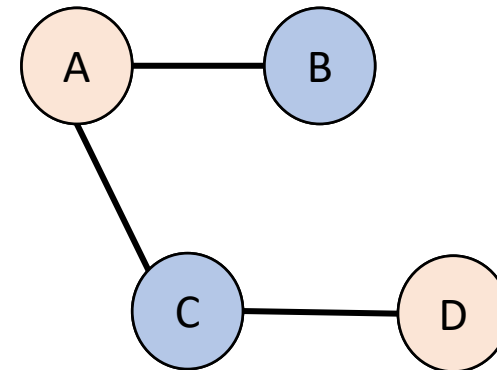
Simplistic Interference Graph:

- Nodes are “variables”
- Edges indicate interference

2-colorable

```
1. [A] := 1
2. [B] := 2
3. output [B]
4. [C] := 3
5. output [A]
6. [D] := 4
7. output [D]
8. output [C]
```

A live: (1, 5)
B live: (2, 3)
C live: (4, 8)
D live: (6, 7)



Recall Data Allocation

SSA – Motivation

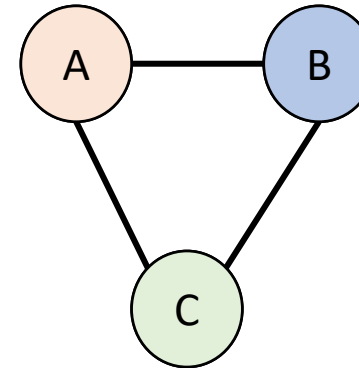
3-colorable

```
1. [A] := 1
2. [B] := 2
3. output [B]
4. [C] := 3
5. output [A]
6. [B] := 4
7. output [B]
8. output [C]
```

A live: (1, 5]

B live: (2, 3] and (6,7]

C live: (4, 8]



Breaking out B into *more* variables uses *fewer* resources!

2-colorable

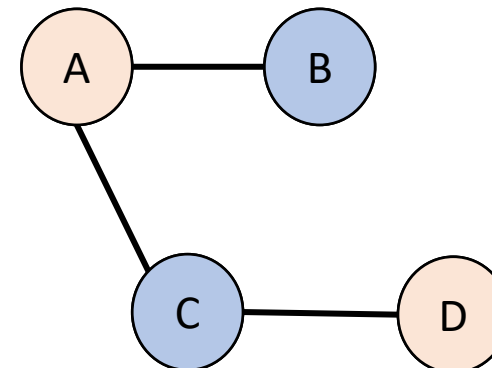
```
1. [A] := 1
2. [B] := 2
3. output [B]
4. [C] := 3
5. output [A]
6. [D] := 4
7. output [D]
8. output [C]
```

A live: (1, 5]

B live: (2, 3]

C live: (4, 8]

D live: (6, 7]



The Static Single Assignment Concept

SSA

An additional restriction on the IR:

- Every variable is assigned a value in *at most one* program point

We can say 3AC is (or isn't) in *SSA form*

Why does that matter?

Disentangles value use
Simplifies other analyses

```
a := 1  
b := a  
c := a + b
```



```
a := 1  
b := a  
a := b * 2  
c := a + b
```



```
L1: b := 7  
goto L1
```



*Ok! statically defined only
once (doesn't matter that it's
dynamically assigned > 1)*

Transformation to SSA Form

SSA

Basic Idea

- Break noncompliant variables into multiple “versions”
- Preserve semantics!

Obvious within a BBL

- Each definition rewritten to a new variable version
- Each use rewritten to the most recently defined variable version

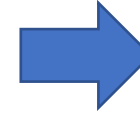
Before
(not SSA form)

$[a] := 1$

$[b] := [a]$

$[a] := [b] * 2$

$[c] := [a] + [b]$



After
(is SSA form)

$[a_1] := 1$

$[b] := [a_1]$

$[a_2] := [b] * 2$

$[c] := [a_2] + [b]$

*quick note on notation:
Ok to leave off the subscript
if there's only one “version”*

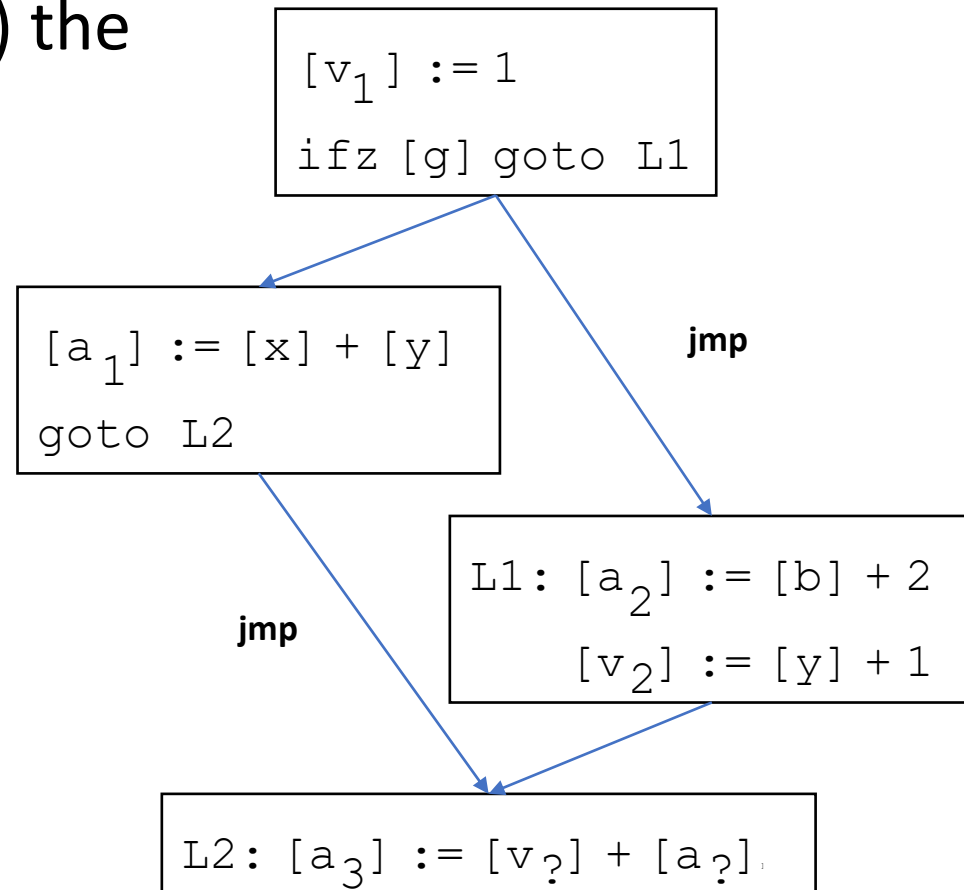
Transformation to SSA Form

SSA

Non-Obvious between BBLs

- Don't know (statically) the most recently defined variable version

```
[v] := 1
ifz [g] goto L1
[a] := [x] + [y]
goto L2
L1: [a] := [b] + 2
    [v] := [y] + 1
L2: [a] := [v] + [a]
```



ϕ Functions – Notational Placeholders

SSA – ϕ Functions

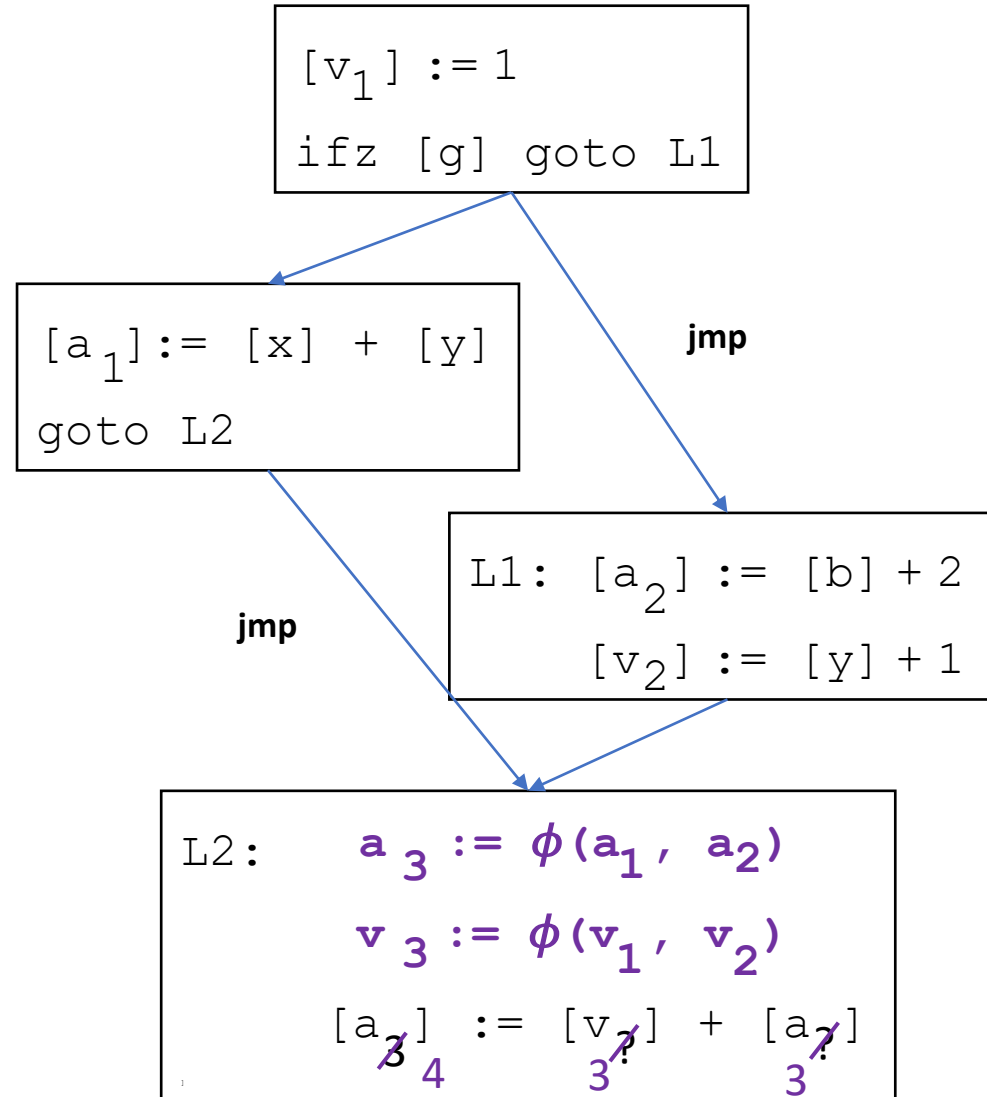
Encapsulated the uncertainty of which version to use

$$a_4 := \phi(a_1, a_2, a_3)$$

means that a_4 will hold whichever version of **a** was defined most recently

ϕ Functions – Resolving “Conflicts”

SSA – ϕ Functions



Example Time – Transform to SSA Form

SSA – ϕ Functions

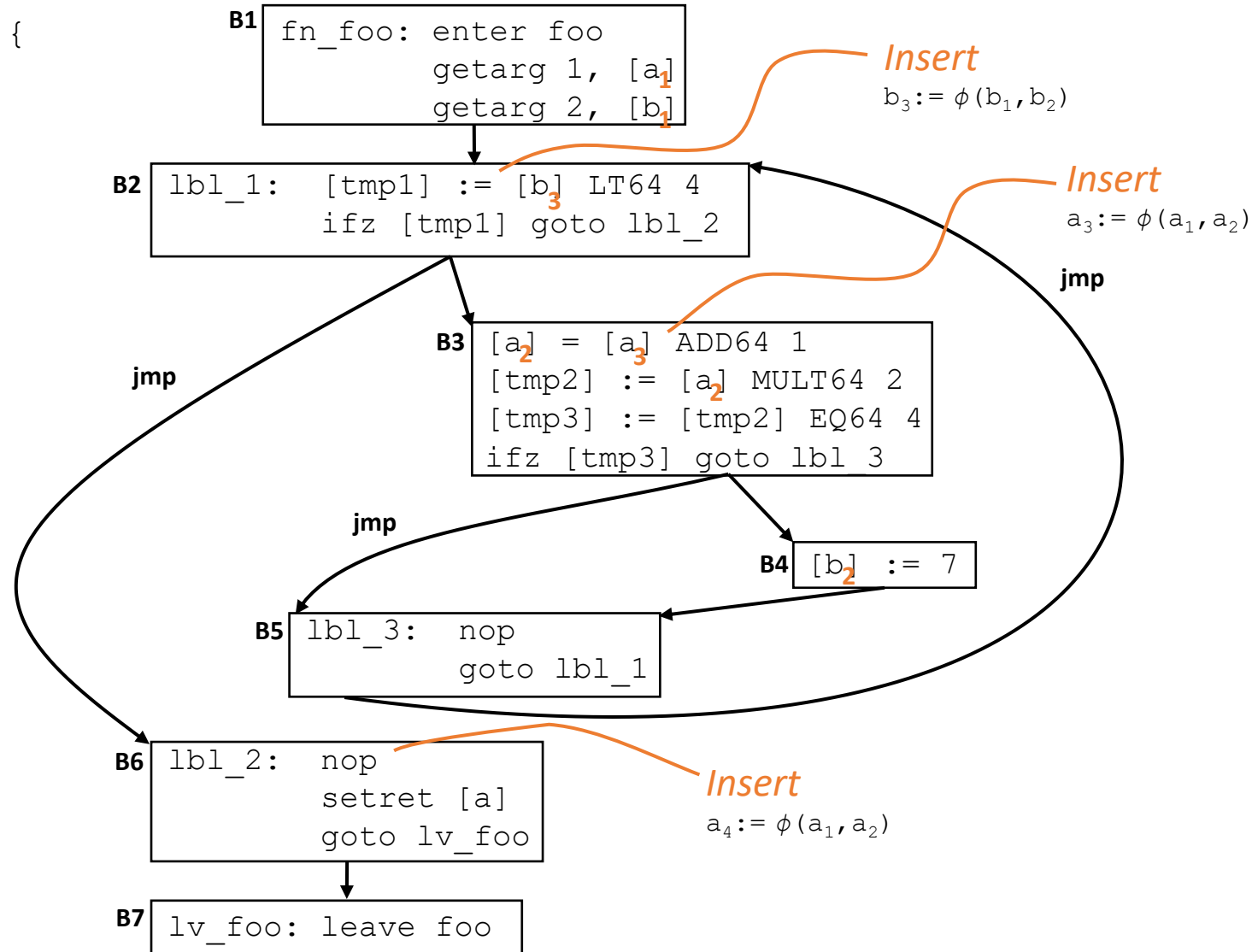
```
int foo(int a, int b){  
    while(b < 4){  
        a += 1;  
        if ( a * 2 == 4){  
            b = 7;  
        }  
    }  
    return a;  
}
```

B1	fn_foo: enter foo getarg 1, [a] getarg 2, [b]
B2	lbl_1: [tmp1] := [b] LT64 4 ifz [tmp1] goto lbl_2
B3	[a] = [a] ADD64 1 [tmp2] := [a] MULT64 2 [tmp3] := [tmp2] EQ64 4 ifz [tmp3] goto lbl_3
B4	[b] := 7
B5	lbl_3: nop goto lbl_1
B6	lbl_2: nop setret [a] goto lv_foo
B7	lv_foo: leave foo

Example Time – Transform to SSA Form

SSA – ϕ Functions

```
int foo(int a, int b){  
    while(b < 4){  
        a += 1;  
        if ( a * 2 == 4){  
            b = 7;  
        }  
    }  
    return a;  
}
```



ϕ Functions – A “Magical” Placeholder

SSA – ϕ Functions

Why rely on a function we cannot compute?

We can remove the ϕ s later

- Easy solution: make sure that all arguments to the ϕ share a common memory location

$a_3 := \phi(a_1, a_2)$

$-24(\%rbp)$ $-24(\%rbp)$

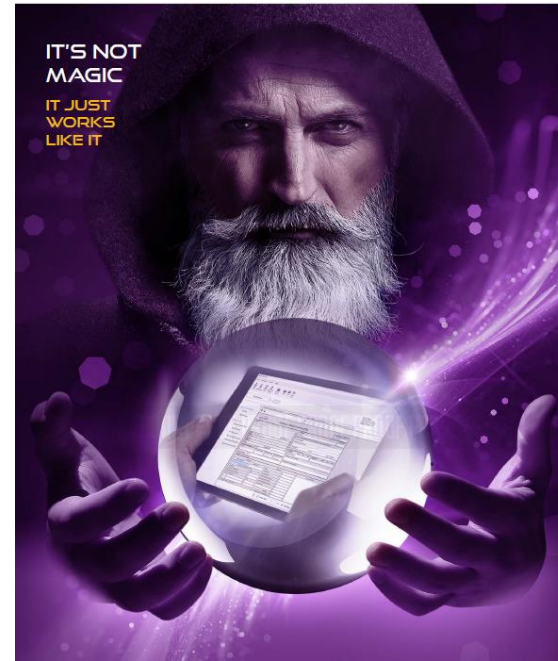


Image Credit: Avyst e-forms wizard

ϕ Functions are Costly!

SSA – Placing ϕ s

Rolls back our sub-variable resource goals

- Consider a naïve algorithm to place ϕ s:
 - Place ϕ for every defined version of the variable

What Points Actually Require ϕ ?

SSA – Placing ϕ s

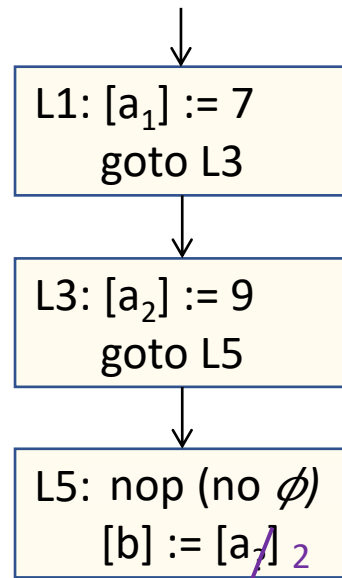
One sufficient condition for Avoiding ϕ nodes:

(wlog, assume Block A defines x and Block B uses x)

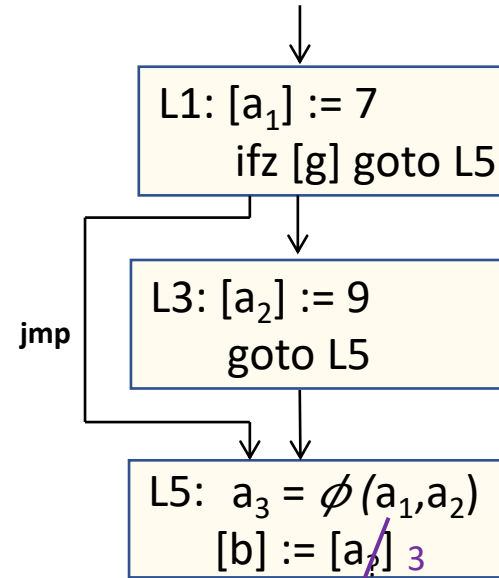
- Block B has an *unambiguous variable definition* if you're guaranteed to go through block A on any path to B

There's a name for this constraint...

Possible CFG Snippet 1



Possible CFG Snippet 2



Domination



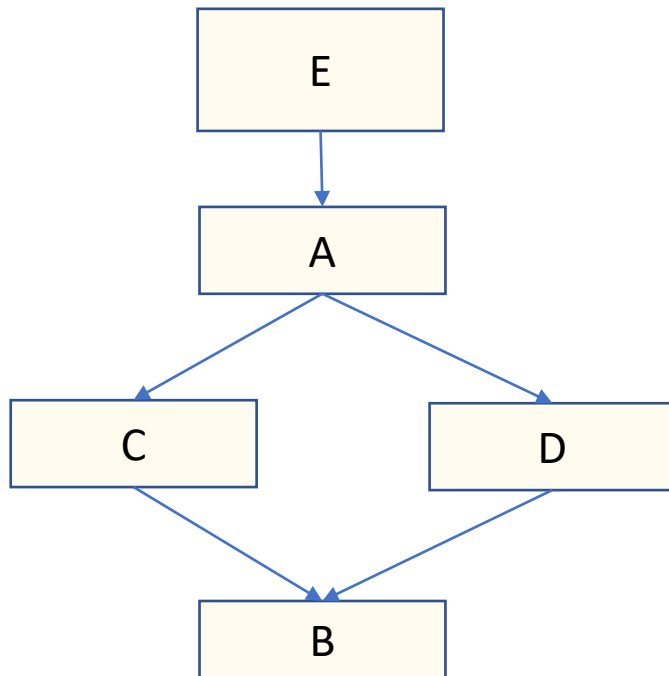
Domination Examples

SSA – Placing ϕ s

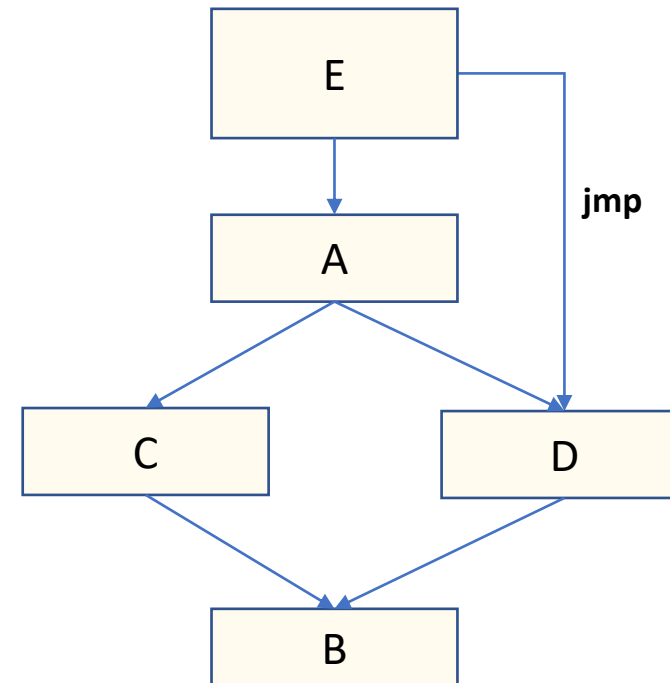
Block X **dominates** block Y if all paths to Y must pass through X

Examples (what does A dominate?)

A **dominates** A, D, C, B



A **dominates** A and C only



Domination Vocabulary

SSA – Placing ϕ s

X DOM Y – *X dominates Y*

- All paths to Y go through X
- (Reflexive - X DOM X)

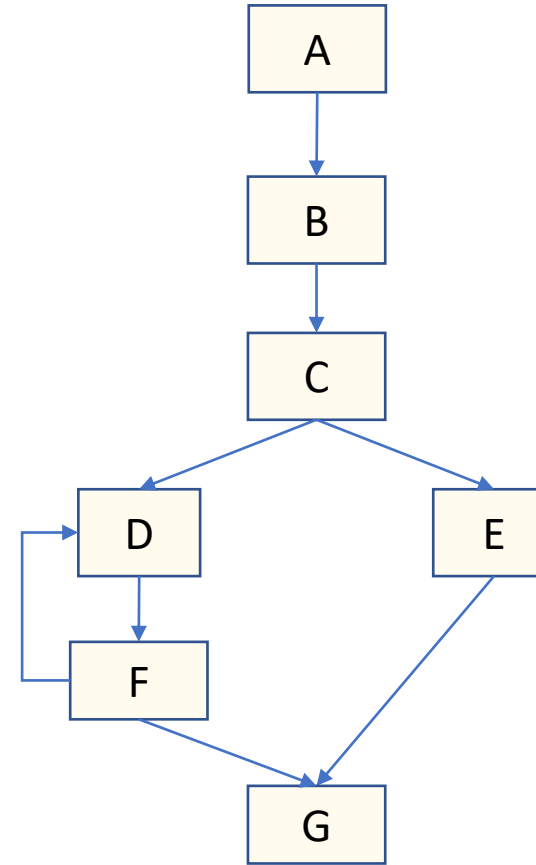
X SDOM Y – *X strictly dominates Y*

- Non-reflexive domination
- Formally: X DOM Y and X \neq Y

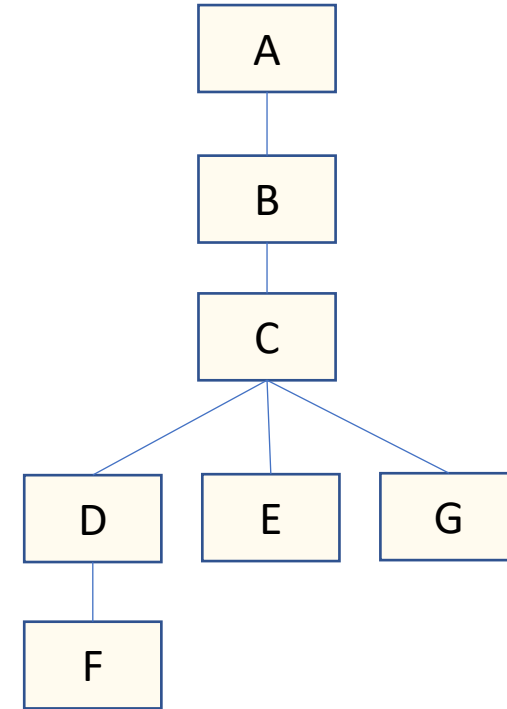
X IDOM Y – *X immediately dominates Y*

- “Closest” strict dominator
- Formally: X SDOM Y and Z SDOM Y \Rightarrow Z = X

Control-Flow Graph



Dominator Tree



What Good is Domination?

SSA – Placing ϕ s

Provides guarantees about execution (sorta-kinda like a looser version of statements being in the same basic block)

- A given block can rely on statements in a dominator to always have happened before the block is executed
- Similarly, a given block cannot rely on statements in non-dominators to always have happened before the block is executed

The boundary has interesting properties for SSA

Wdetour: Using Dominators for ϕ s

SSA – Placing ϕ s



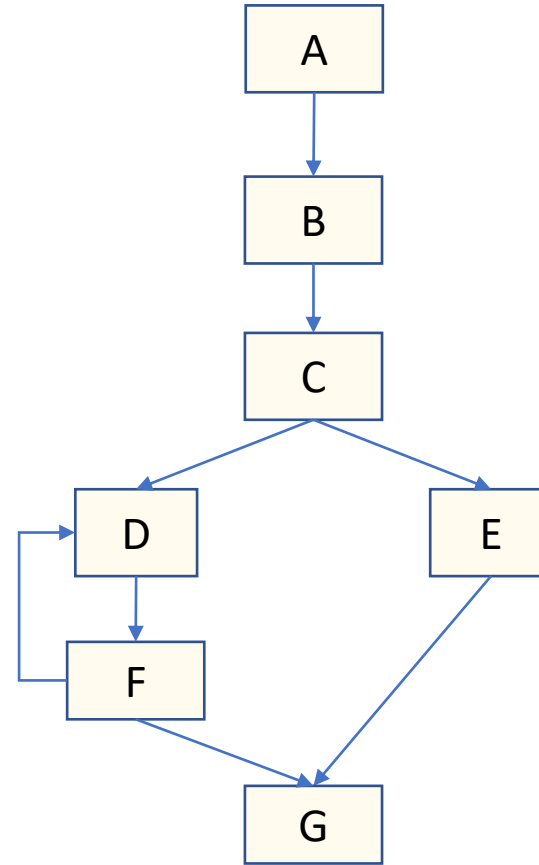
Domination Vocabulary

SSA – Placing ϕ s



Dominator Frontier of X:

The set of nodes k_i
that X does not strictly dominate,
but X dominates an immediate
predecessor of k_i



Example Time – Compute Dom Frontier

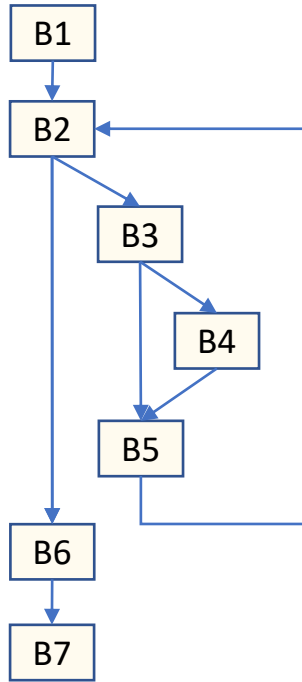
SSA – ϕ Functions



B1 What does B1 dominate? B1 B2 B3 B4 B5 B6 B7
 What do these precede? ~~B2~~ ~~B3~~ ~~B6~~ ~~B4~~ ~~B5~~ ~~B2~~ ~~B7~~
 Disqualify if B1 SDOMs

Dominator Frontier of X:

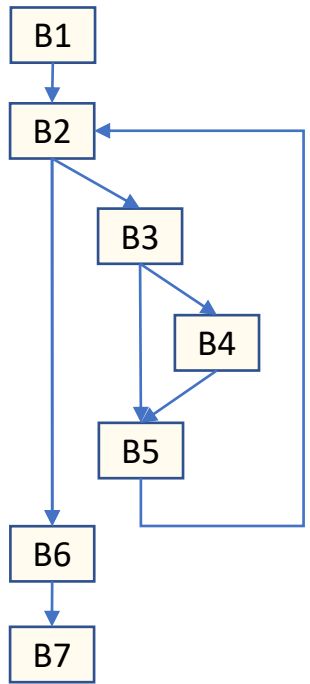
The set of nodes k_i
 $\neg X \text{ SDOM } k_i$
 $X \text{ DOM } Y \text{ and } Y \text{ IPRED } k_i$



BBL	IPRED	DOM	SDOM	DF
B1	B2	(all)	B2,B3,B4,B5,B6,B7	{}
B2	B3, B6	B2,B3,B4,B5,B6,B7	B3,B4,B5,B6,B7	
B3	B4,B5	B3, B4,B5	B4,B5	
B4	B5	B4	{}	
B5	B2	B5	{}	
B6	B7	B6,B7	B7	
B7	{}	B7	{}	

Example Time – Compute Dom Frontier

SSA – ϕ Functions



Dominator Frontier of X:

The set of nodes k_i

! X SDOM k_i

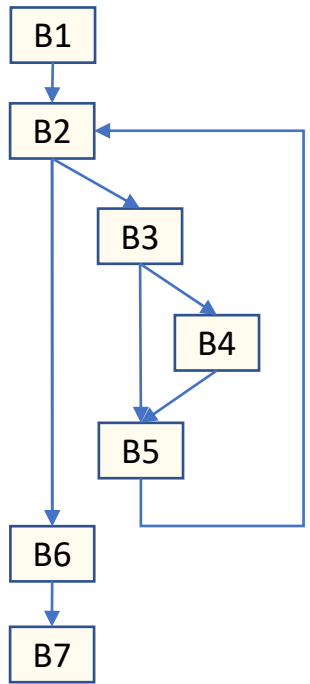
X DOM Y and Y IPRED k_i

BBL	IPRED	DOM	SDOM	DF
B1	B2	(all)	B2,B3,B4,B5,B6,B7	{}
B2	B3, B6	B2,B3,B4,B5,B6,B7	B3,B4,B5,B6,B7	B2
B3	B4,B5	B3, B4,B5	B4,B5	B2
B4	B5	B4	{}	
B5	B2	B5	{}	
B6	B7	B6,B7	B7	
B7	{}	B7	{}	

- B1 What does B1 dominate? B1 B2 B3 B4 B5 B6 B7
 What do these precede? ~~B2~~ ~~B3~~ ~~B6~~ ~~B4~~ ~~B5~~ ~~B2~~ ~~B7~~
 Disqualify if B1 SDOMs
- B2 What does B2 dominate? B2 B3 B4 B5 B6 B7
 What do these precede? ~~B3~~ ~~B6~~ ~~B4~~ ~~B5~~ ~~B5~~ ~~B2~~ ~~B7~~
 Disqualify if B2 SDOMs
- B3 What does B3 dominate? B3 B4 B5
 What do these precede? ~~B4~~ ~~B5~~ ~~B5~~ B2
 Disqualify if B3 SDOMs

Example Time – Compute Dom Frontier

SSA – ϕ Functions



Dominator Frontier of X:

The set of nodes k_i

! X SDOM k_i

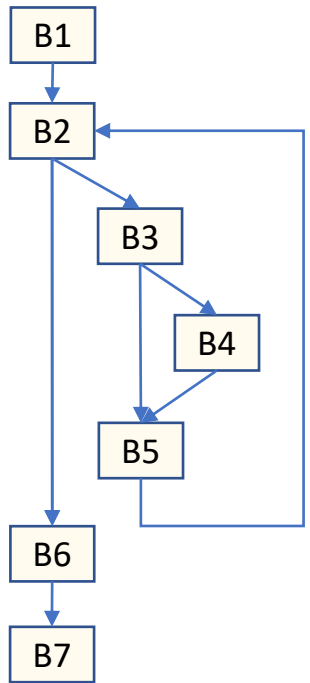
X DOM Y and Y IPRED k_i

BBL	IPRED	DOM	SDOM	DF
B1	B2	(all)	B2,B3,B4,B5,B6,B7	{}
B2	B3, B6	B2,B3,B4,B5,B6,B7	B3,B4,B5,B6,B7	B2
B3	B4,B5	B3, B4,B5	B4,B5	B2
B4	B5	B4	{}	B5
B5	B2	B5	{}	B2
B6	B7	B6,B7	B7	
B7	{}	B7	{}	

- B1 What does B1 dominate? B1 B2 B3 B4 B5 B6 B7
 What do these precede? ~~B2~~ ~~B3~~ ~~B6~~ ~~B4~~ ~~B5~~ ~~B2~~ ~~B7~~
 Disqualify if B1 SDOMs
- B2 What does B2 dominate? B2 B3 B4 B5 B6 B7
 What do these precede? ~~B3~~ ~~B6~~ ~~B4~~ ~~B5~~ ~~B5~~ ~~B2~~ ~~B7~~
 Disqualify if B2 SDOMs
- B3 What does B3 dominate? B3 B4 B5
 What do these precede? ~~B4~~ ~~B5~~ ~~B5~~ B2
 Disqualify if B3 SDOMs
- B4 What does B4 dominate? B4
 What do these precede? B5
 Disqualify if B4 SDOMs
- B5 What does B5 dominate? B5
 What do these precede? B2
 Disqualify if B5 SDOMs

Example Time – Compute Dom Frontier

SSA – ϕ Functions



Dominator Frontier of X:

The set of nodes k_i

! X SDOM k_i

X DOM Y and Y IPRED k_i

BBL	IPRED	DOM	SDOM	DF
B1	B2	(all)	B2,B3,B4,B5,B6,B7	{}
B2	B3, B6	B2,B3,B4,B5,B6,B7	B3,B4,B5,B6,B7	B2
B3	B4,B5	B3, B4,B5	B4,B5	B2
B4	B5	B4	{}	B5
B5	B2	B5	{}	B2
B6	B7	B6,B7	B7	{}
B7	{}	B7	{}	{}

B1 What does B1 dominate? B1 B2 B3 B4 B5 B6 B7
 What do these precede? ~~B2~~ ~~B3~~ ~~B6~~ ~~B4~~ ~~B5~~ ~~B2~~ ~~B7~~
 Disqualify if B1 SDOMs

B2 What does B2 dominate? B2 B3 B4 B5 B6 B7
 What do these precede? ~~B3~~ ~~B6~~ ~~B4~~ ~~B5~~ ~~B5~~ B2 ~~B7~~
 Disqualify if B2 SDOMs

B3 What does B3 dominate? B3 B4 B5
 What do these precede? ~~B4~~ ~~B5~~ ~~B5~~ B2
 Disqualify if B3 SDOMs

B4 What does B4 dominate? B4
 What do these precede? B5 B7
 Disqualify if B4 SDOMs

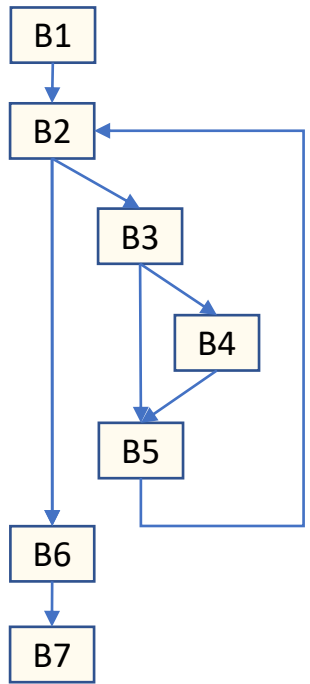
What does B7 dominate? B7
 What do these precede? {}

B5 What does B5 dominate? B5
 What do these precede? B2
 Disqualify if B5 SDOMs

B6 What does B6 dominate? B6 B7
 What do these precede? ~~B7~~
 Disqualify if B6 SDOMs

Example Time – Compute Dom Frontier

SSA – ϕ Functions



Dominator Frontier of X:

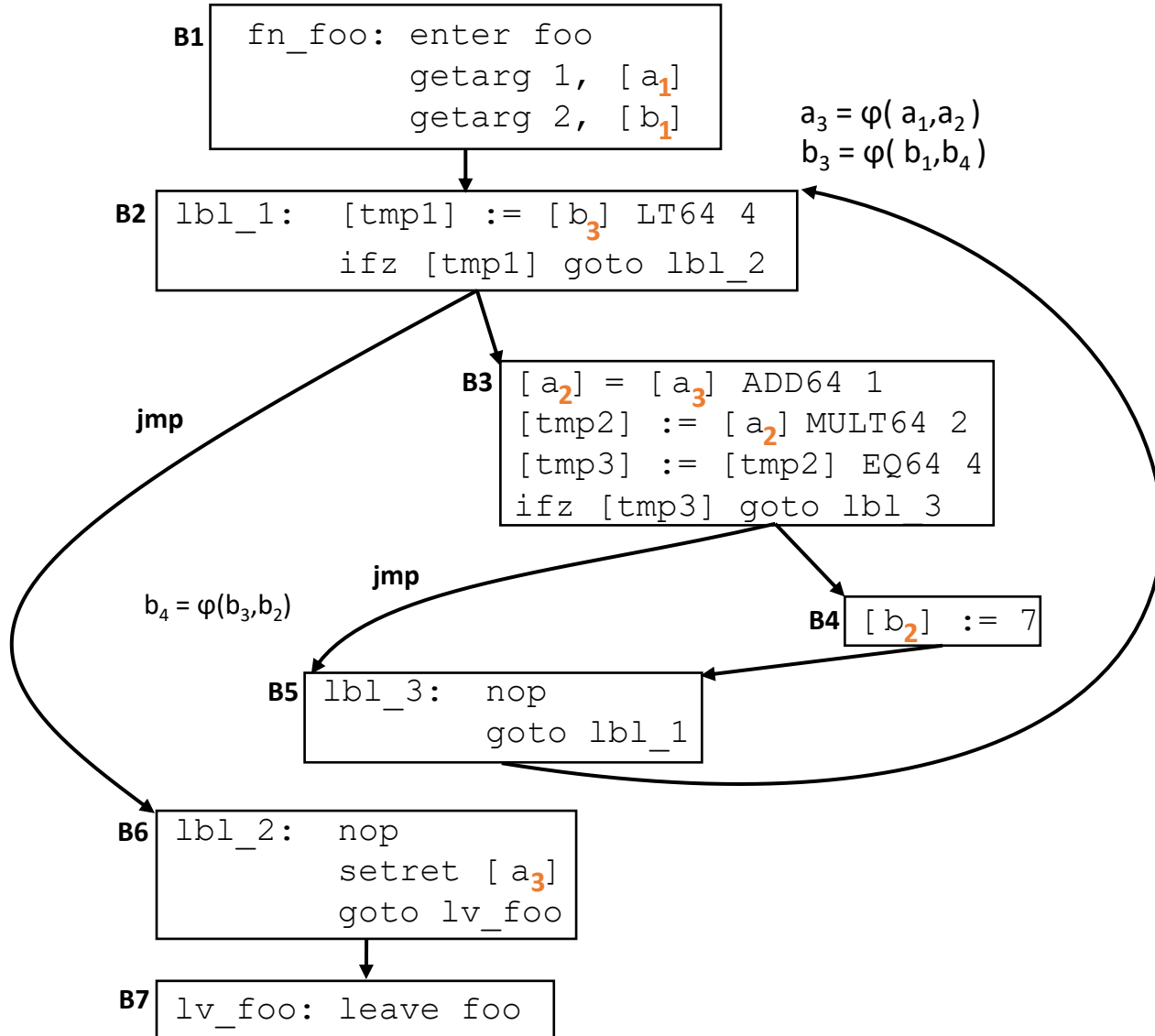
The set of nodes k_i
! X SDOM k_i
X DOM Y and Y IPRED k_i

```
for v in vars:
    for d in DefBBLs[v]:
        for block in DF[d]:
            Add a  $\phi$ -node to block,
            unless we have done so already.
        Add block to DefBBLs[v]
        unless it's already in there.
```

BBL	IPRED	DOM	SDOM	DF
B1	B2	(all)	B2,B3,B4,B5,B6,B7	{}
B2	B3, B6	B2,B3,B4,B5,B6,B7	B3,B4,B5,B6,B7	B2
B3	B4,B5	B3, B4,B5	B4,B5	B2
B4	B5	B4	{}	B5
B5	B2	B5	{}	B2
B6	B7	B6,B7	B7	{}
B7	{}	B7	{}	{}

Example Time – Compute Dom Frontier

SSA – ϕ Functions



```

for v in vars:
    for d in DefBBLs[v]:
        for block in DF[d]:
            Add a  $\phi$ -node to block,
            unless we have done so already.
            Add block to DefBBLs[v]
            unless it's already in there.
  
```

var	DefBBLs	Φ Blocks
a	B1 B3 B2	B2
b	B1 B4 B5 B2	B5 B2

BBL	IPRED	DOM	SDOM	DF
B1	B2	(all)	B2,B3,B4,B5,B6,B7	{}
B2	B3, B6	B3,B4,B5,B6,B7	B3,B4,B5,B6,B7	B2
B3	B4,B5	B3, B4,B5	B4,B5	B2
B4	B5	B4	{}	B5
B5	B2	B5	{}	B2
B6	B7	B6,B7	B7	{}
B7	{}	B7	{}	{}

End Detour: Using Dominators for ϕ s

SSA – Placing ϕ s



Dominance: Summary

SSA – Placing ϕ s

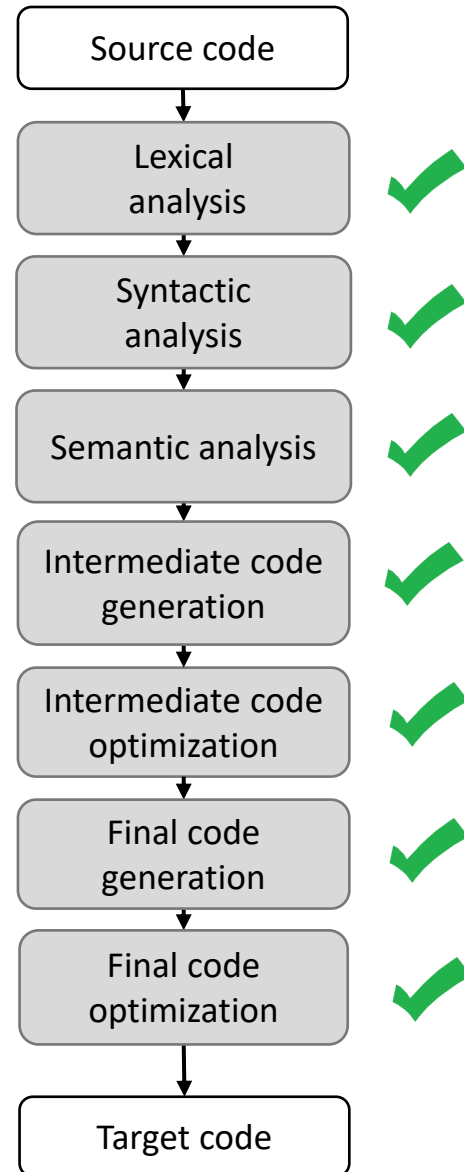
Summary:

- Dominators can be computed efficiently
- Dominance can be used to aid in efficient SSA
- SSA aids in efficient program optimization and future analysis



Oh Hey, We Built a Compiler!

Underview





What Next?

Underview

Practical Applications

Why does this class matter?

- “So you can do compilers”: Practical skills for language implementation / reasoning
- “What you do with compilers is useful outside doing compilers”