

Build Selector Table

Review: LL(1) SDT

Build the selector table for this grammar

Grammar

- ① $S ::= \text{lpar } X \text{ rpar}$
- ② $X ::= \text{id comma } X$
- ③ $X ::= \epsilon$

$\text{FIRST}(S) = \{ \text{lpar} \}$

$\text{FIRST}(X) = \{ \epsilon, \text{id} \}$

$\text{FOLLOW}(S) = \{ \text{eof} \}$

$\text{FOLLOW}(X) = \{ \text{rpar} \}$

Building Selector Tables

for each production $X ::= \alpha$

for each t in $\text{FIRST}(\alpha)$

put $X ::= \alpha$ in $\text{Table}[X][t]$

if ϵ is in $\text{FIRST}(\alpha)$

for each t in $\text{FOLLOW}(X)$

put $X ::= \alpha$ in $\text{Table}[X][t]$

$\text{FIRST}(\text{lpar } X \text{ rpar}) = \{ \text{lpar} \}$
 $\text{FIRST}(\text{id comma } X) = \{ \text{id} \}$

	lpar	id	comma	rpar	eof
S	① lpar X rpar				
X		② id comma X		③ ϵ	

~~lpar~~
~~id~~
~~comma~~
~~rpar~~
~~eof~~

~~X~~

~~lpar~~

~~id~~ ~~comma~~ ~~rpar~~ eof

Administrivia

Housekeeping

Thanks to Andrew for the guest lecture!

- His slides / video are now posted to the class website

Guest Lecture Question

Housekeeping

CFG

$Expr ::= \epsilon \#1$
 $\quad | (Expr) \#2$
 $\quad | [Expr] \#3$

SDT Actions: Counting Max Parens Depth

$\#1$ push 0
 $\#2$ ExprTrans = pop; push(ExprTrans + 1)
 $\#3$ ExprTrans = pop; push(ExprTrans)

([]) EOF

$Expr$
 $\{$
 $Expr$
 $\}$
 $\#3$

(Expr) #2	[Expr] #3	ϵ #1	ϵ #1	ϵ #1
-------------	-------------	---------------	---------------	---------------

~~Expr~~
 eof

t_i i[
 lookahead

$[()]$
 $\uparrow$
 t_i



EECS 665

COMPIILER

CONSTRUCTION

LR Parsing

Last Time

Review LL(1) SDT

LL(1) SDT

- Add 2nd stack for symbol translations
- Turn SDD to semantic stack actions
- Embed triggers into syntactic stack

You should know

How to convert from SDD to stack actions

How to run SDT for an LL(1) parser



Parsing

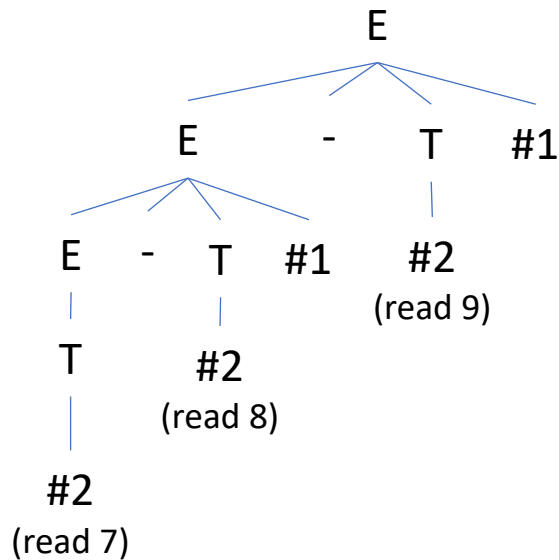
Actions Preserved Through Transforms

3 Grammars for Subtraction Expression:

intlit(7) – intlit(8) – intlit(9)

Augmented Left-Assoc CFG

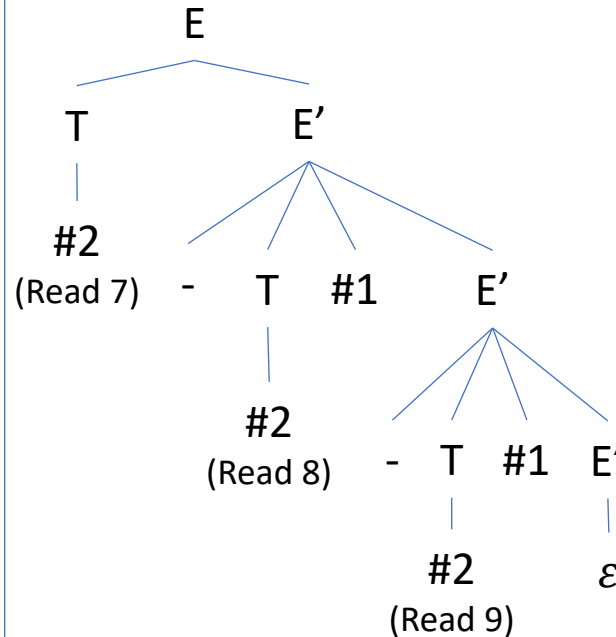
$E ::= E - T \#1$
| T
 $T ::= \#2 \text{ intlit}$



#2 #2 #1 #2 #1
7 8 -1 9 -10

LL(1) CFG

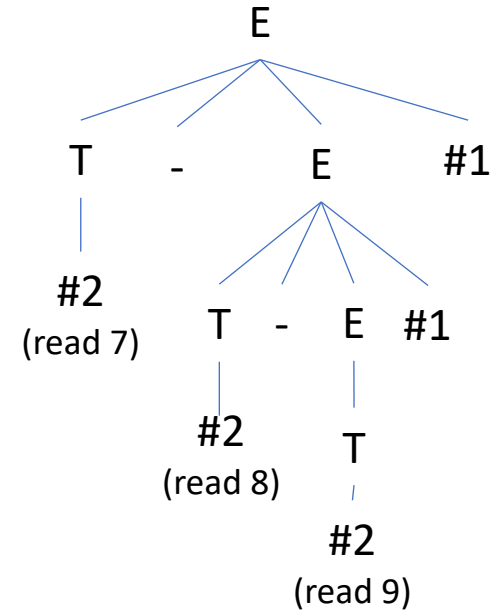
$E ::= T E'$
 $E' ::= - T \#1 E' \mid \epsilon$
 $T ::= \#2 \text{ intlit}$



#2 #2 #1 #2 #1
7 8 -1 9 -10

Augmented Right-Assoc CFG

$E ::= T - E \#1$
| T
 $T ::= \#2 \text{ intlit}$



#2 #2 #2 #1 #1 -8
7 8 9 -1 -10 6

Today's Outline

Lecture 11 – LR Parsing

LL(1) Wrap-up

- Limitations

LR Parsers

- Concept
- Theory
- Operation



Parsing

LL(1) Summary

LL(1) Wrap-Up

Predictive Parser

- Commits to an entire production based on observed lookahead. For LL(1), predict a production based on 1 token of lookahead

Implementation

- Stack and a single streamable token

Many Useful Languages are NOT LL(1)

LL(1) Wrap-Up

Could increase lookahead

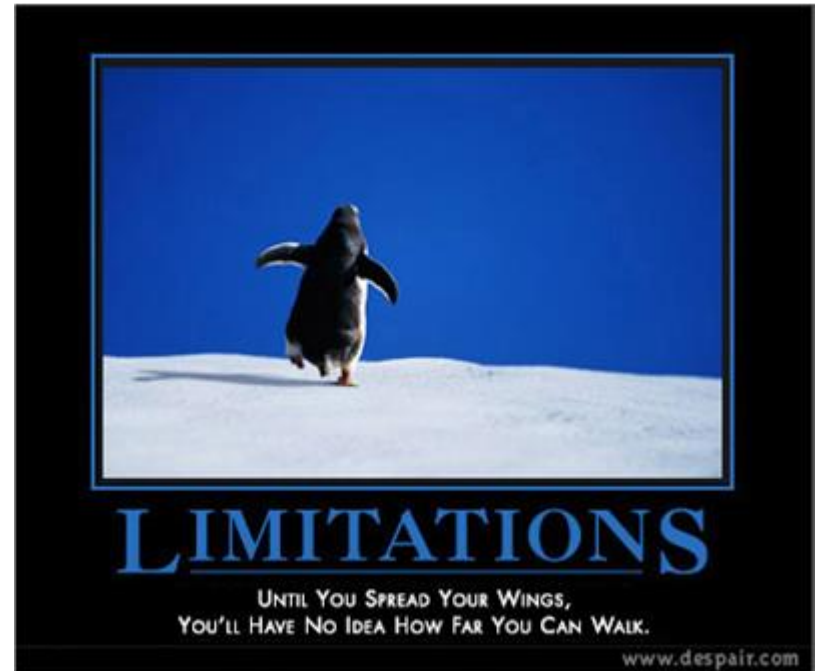
- Up until the mid 90s, this was considered impractical

Could add runtime complexity

- CYK has us covered there

Could add memory complexity

- i.e. more elaborate parse table



LR Parsers

LR Parsers - Concept

Advantages

- Suitable for most programming languages
- Runtime: time and space $O(n)$ in the input size
- Subsumes corresponding LL parser i.e. $LL(1) < LR(1)$
- $LR(1)$ parsers can recognize any Deterministic CFG

Disadvantages

- More complex parser generation
- Larger memory state

Linear Parsers: Commitment Issues

LR Parsers - Concept

Linear Parser have to commit to their parse tree in a linear scan

LL(1) Parser:
commits ASAP



Backtracking Parsers:
Go back on commitments



LR Parsers:
Delay commitments



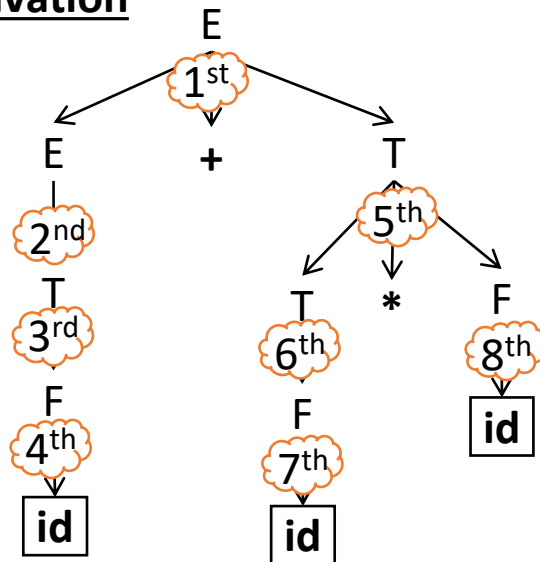
Delayed Commitment to a Rule

LR Parsers - Concept

Can we choose a production *after* we've seen it's subtree?

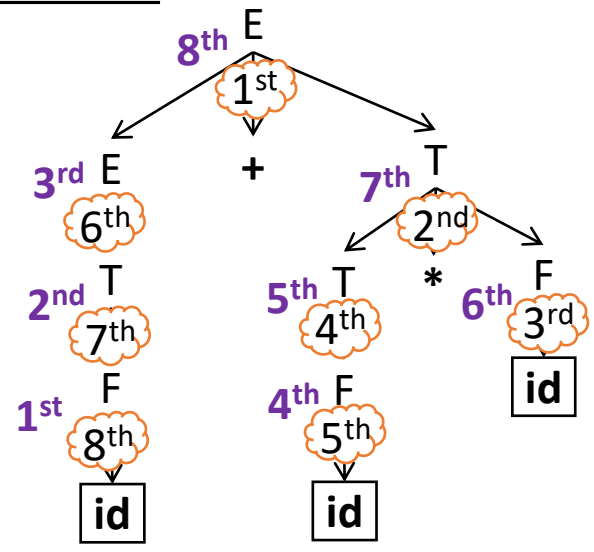
- Yes, if we build the tree *bottom-up*!
- What about derivation order?

Leftmost Derivation



Reverse

Rightmost Derivation



LR(k) Parsers

LR Parsers - Concept

Perform reverse rightmost derivation

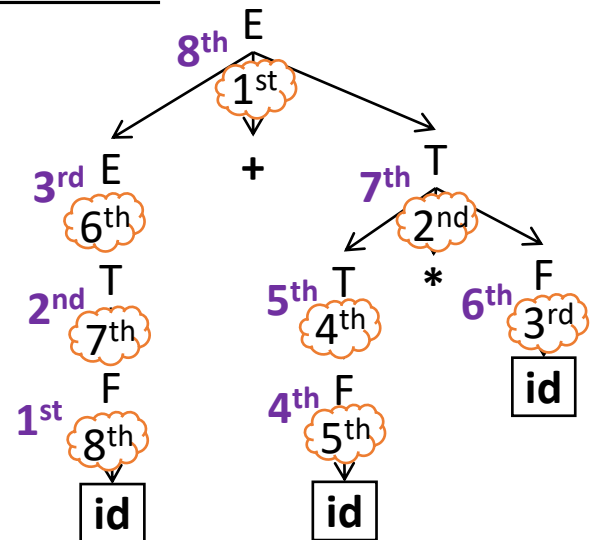
- Always move up branch before starting new branch
- Work on the LAST SYMBOL(S) of the derivation PREFIX

Reverse Rightmost derivation

$E \Rightarrow E + T$
 $\Rightarrow E + T * F$
 $\Rightarrow E + T * id$
 $\Rightarrow E + F * id$
 $\Rightarrow E + id * id$
 $\Rightarrow T + id * id$
 $\Rightarrow F + id * id$
 $\Rightarrow id + id * id$

Reverse

Rightmost Derivation



LR(k) Parsers

LR Parsers - Concept

Perform reverse rightmost derivation

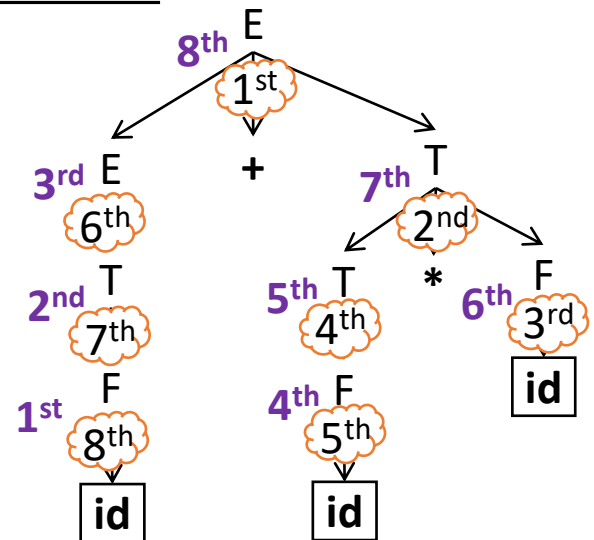
- Always move up branch before starting new branch
- Work on the LAST SYMBOL(S) of the derivation PREFIX

Reverse Rightmost derivation

id| + id * id
F| + id * id
T| + id * id
E + id| * id
E + F| * id
E + T * id|
E + T * F|
E + T| ← E

Reverse

Rightmost Derivation



LL vs LR Parsers

Beyond LL(1)

LL Operations

- *Predict* the rule used to expand nonterminal
- *Verify* that a terminal was expected

LR Operations

- *Shift* to a new branch (add a new leaf)
- *Reduce* a string of symbols to the parent nonterminal

Bottom-Up Parsing – Example

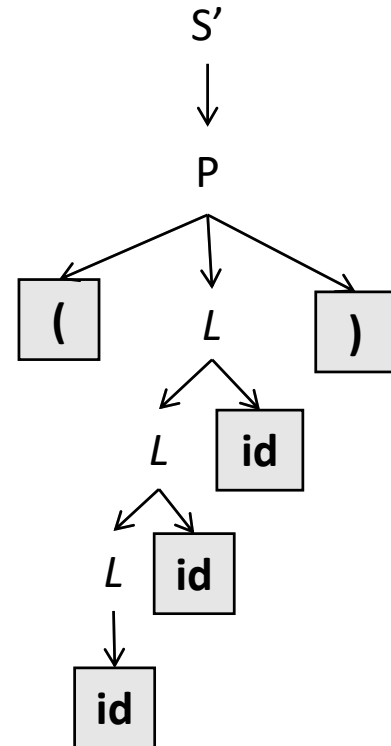
LR Parser Construction

<u>Prefix</u>	<u>Suffix</u>	<u>Action</u>
	(id id id) eof	shift (
(	id id id) eof	shift id
(id	id id) eof	Red. ③ $L ::= id$
(L	id id) eof	shift id
(L id	id) eof	Red. ④ $L ::= L id$
(L	id) eof	shift id
(L id	) eof	Red. ④ $L ::= L id$
(L	) eof	shift)
(L)	eof	
P	eof	Red. ② $L ::= (L)$
S'	eof	Red. ① $S' \rightarrow P$

Accept!!

- Grammar G
- ① $S' ::= P$
 - ② $P ::= (L)$
 - ③ $L ::= id$
 - ④ $L ::= L id$

Correct Parse Tree

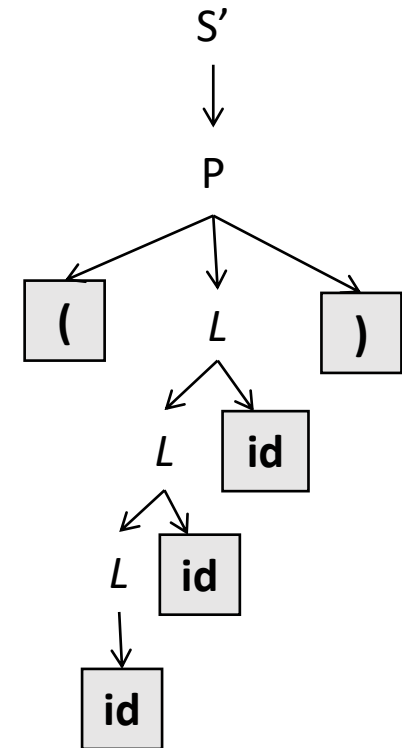


LR Parser – Towards Implementation

LR Parser Construction

Key Points

- Relies on *shift* and *reduce* actions
 - Shift – add a **terminal** leaf to parse tree
 - Reduce – add nonterminal to parse tree
- Builds the parse tree *bottom-up*
 - *Does as many reduce actions as possible before a shift*

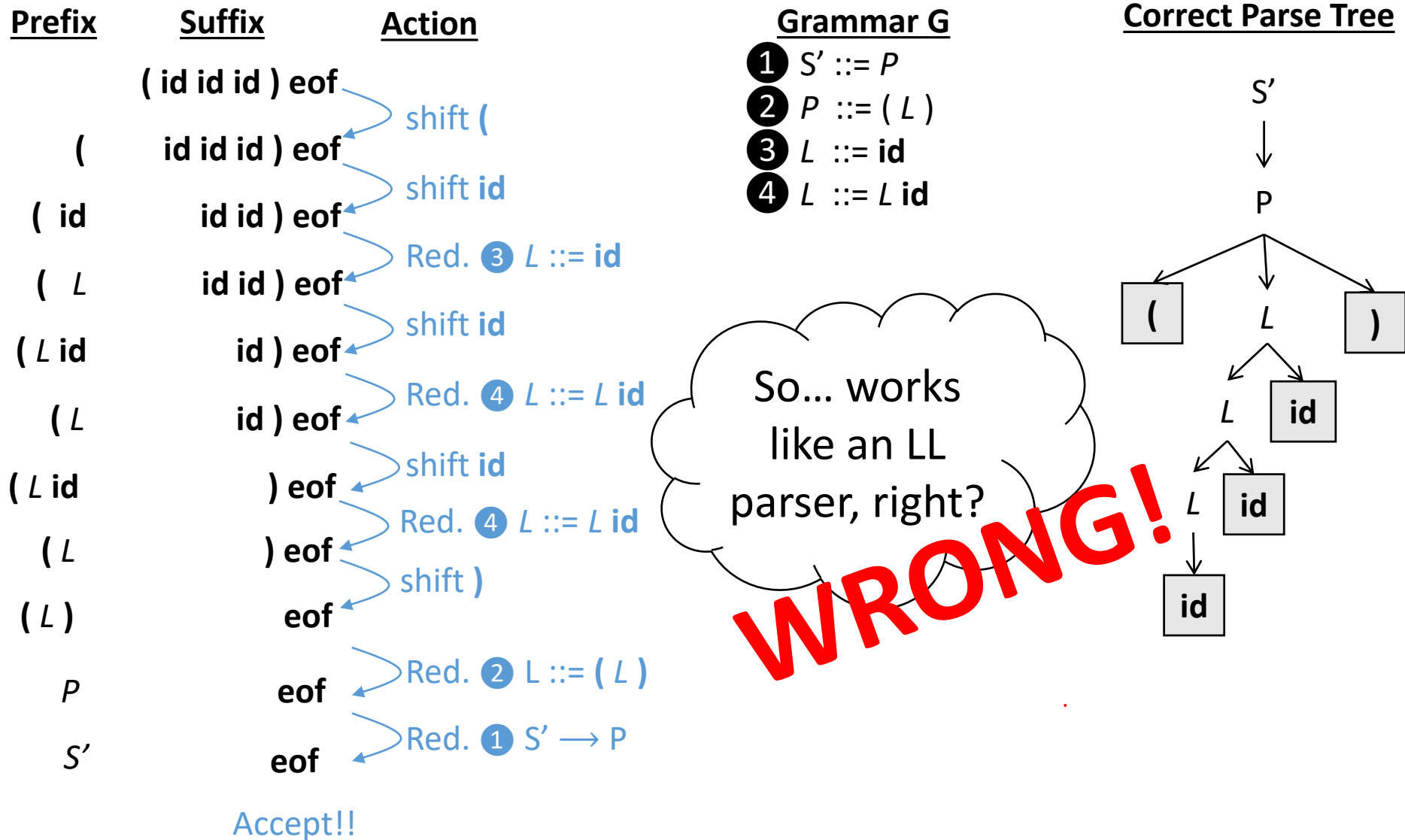


Grammar G

- 1 $S' ::= P$
- 2 $P ::= (L)$
- 3 $L ::= id$
- 4 $L ::= L id$

Bottom-Up Parsing – Implementation

LR Parser Construction



LL vs LR Parser – We are Not the Same

LR Parser Construction

LL(1) - Uses 1 token of lookahead and 1 nonterminal to pick production

“Oh, I’ll just use that same info for the LR(1) parser”



LR Parser – Towards Implementation

My darkest timeline fanfic:

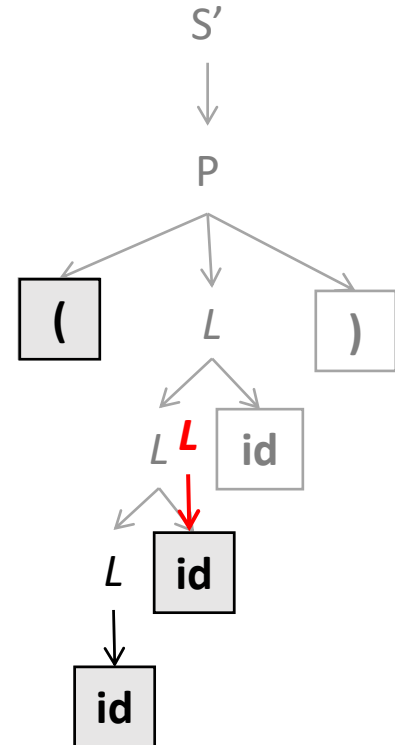
Context: 1 grammar symbols x 1 lookahead

Prefix	Suffix	Action
(	(id id id) eof	shift (
(id	id id id) eof	shift id
(L	id id) eof	Red. ③ $L ::= id$
(L id	id id) eof	shift id
(L L	id) eof	Red. ③ $L ::= L id$
(L L	id) eof	

Grammar G

- 1 $S' ::= P$
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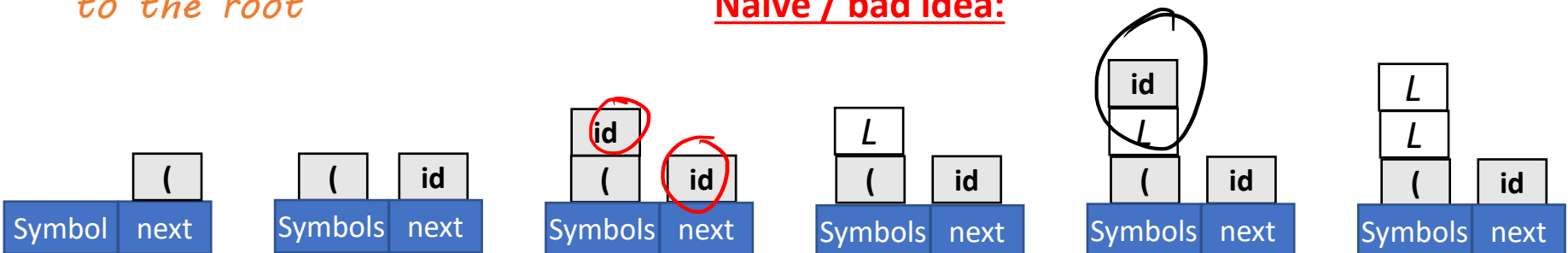
Correct Parse Tree



These are
“viable prefixes”:
They can reduce
to the root

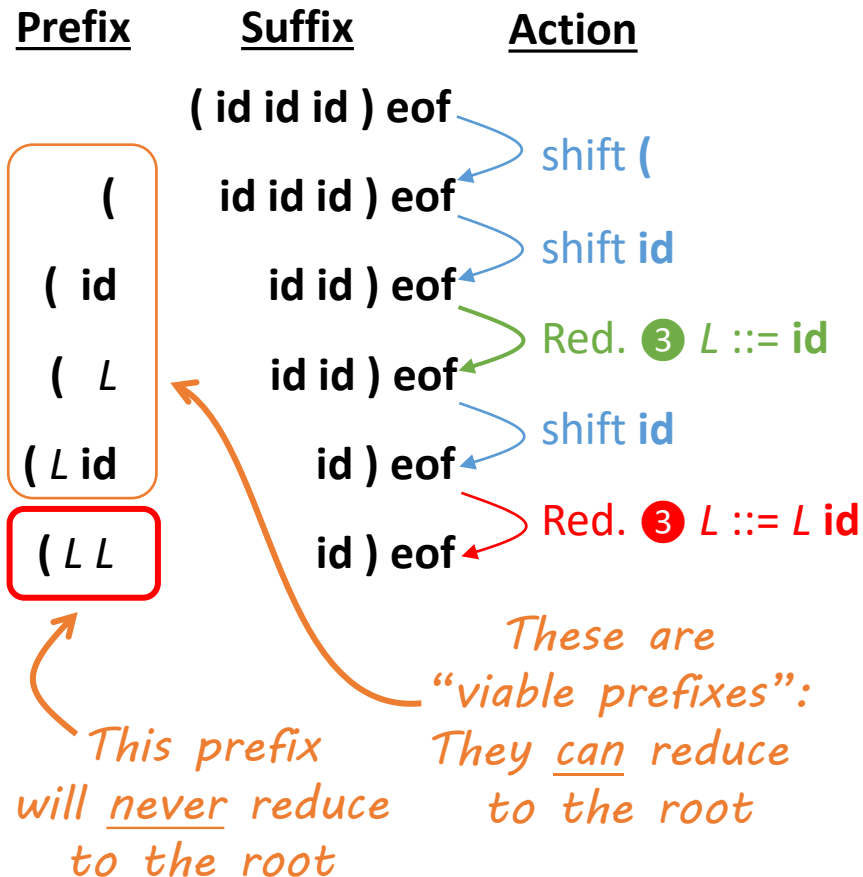
This prefix
will never reduce
to the root

Naïve / bad idea:



LR Parser – Towards Implementation

LR Parser Construction



The parser must maintain a viable prefix

- Ensures no misstep taken
- How to capture the (infinite) viable prefixes?

AMAZING Fact:

- For a deterministic CFG the viable prefixes form a regular language
 - So there's a DFA for it!

LR Parser – Towards Implementation

LR Parser Construction

Prefix	Suffix	Action
.	(id id id) eof	shift (
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These are “viable prefixes”: They can reduce to the root

This prefix will never reduce to the root

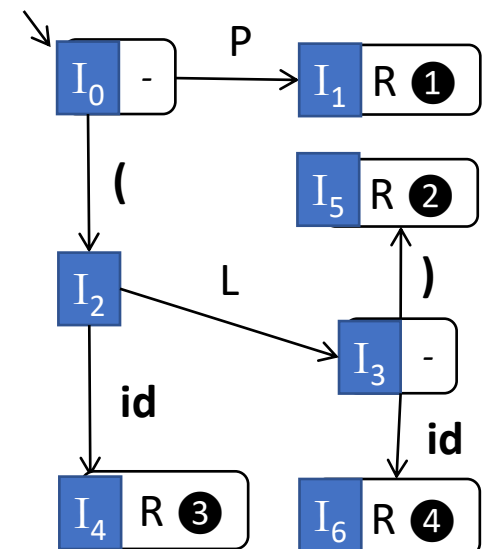
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- For a deterministic CFG the viable prefixes form a regular language
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Grammar G

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G Parser Automaton



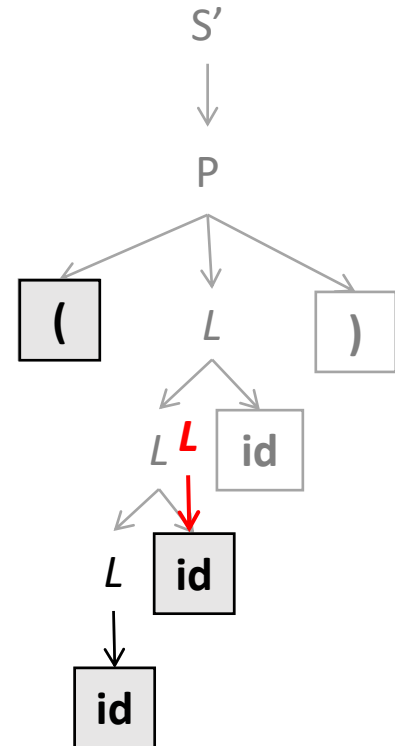
LR Parser – Towards Implementation

LR Parser Construction

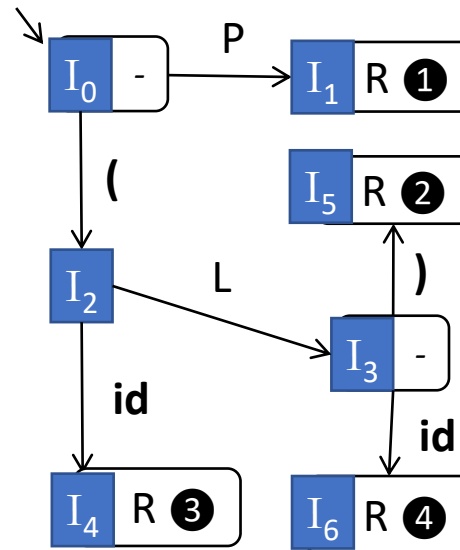
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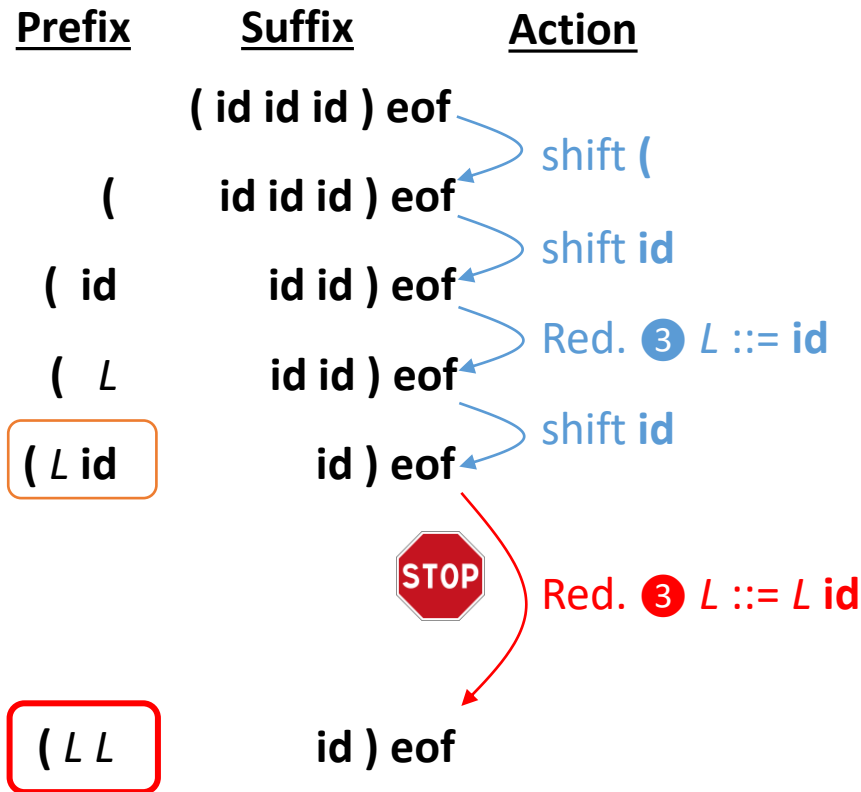


G Parser Automaton



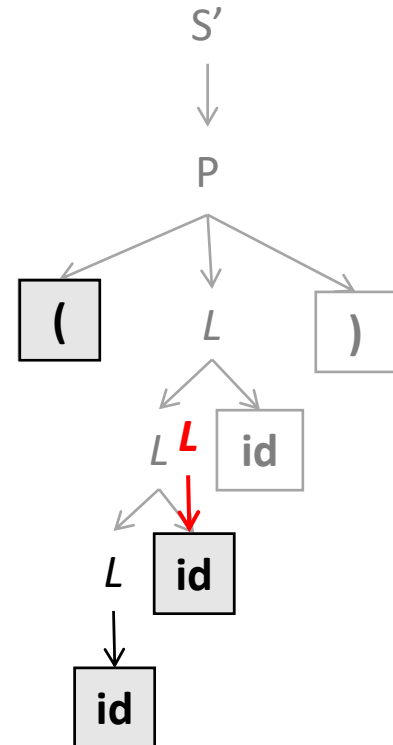
LR Parser – Towards Implementation

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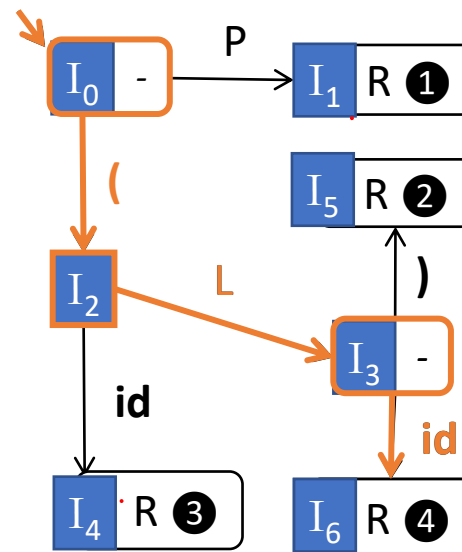


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Correct Parse Tree



G Parser Automaton



LR Parser – Towards Implementation

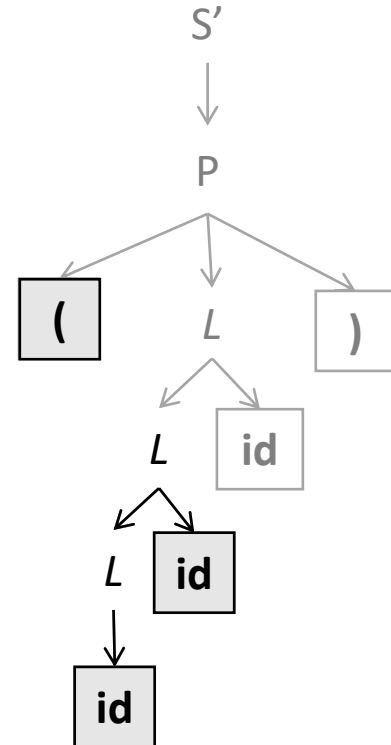
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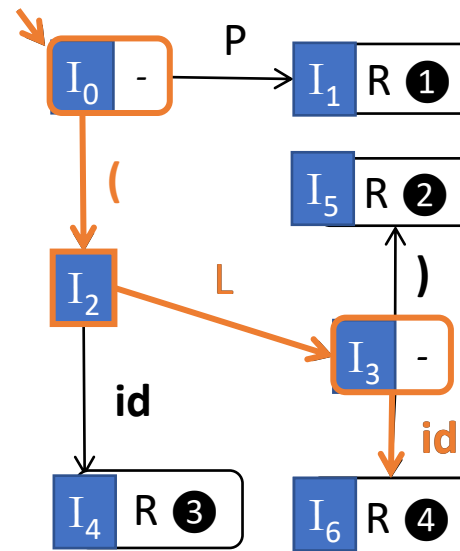
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Correct Parse Tree



G Parser Automaton



Viable Prefix Summary

LR Parser Construction

Any DCFG has a regular language of viable prefixes

- We can use the DFA for that language to prevent missteps

Advancing the automaton

- Terminal Edges – add a terminal to the prefix
- Nonterminal Edges – add nonterminal to the prefix
- Accepting states – modify the existing prefix

Key Idea: Run the DFA for viability

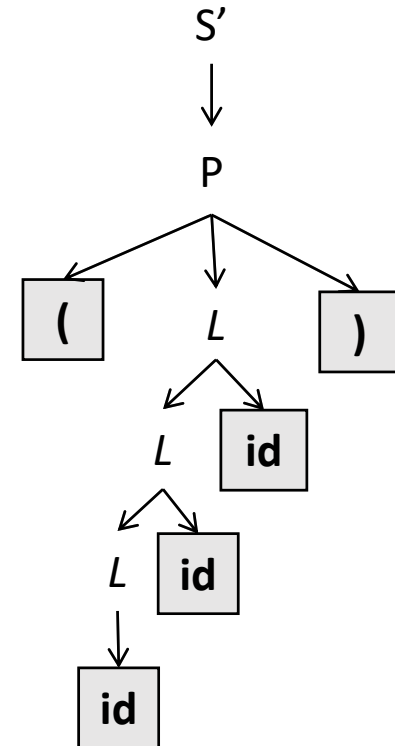
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(L	id) eof	shift id
(L id	) eof	Red. ④ $L ::= L id$
(L	) eof	shift)
(L)	eof	Red. ② $L ::= (L)$
P	eof	Red. ① $S' \rightarrow P$
S'	eof	

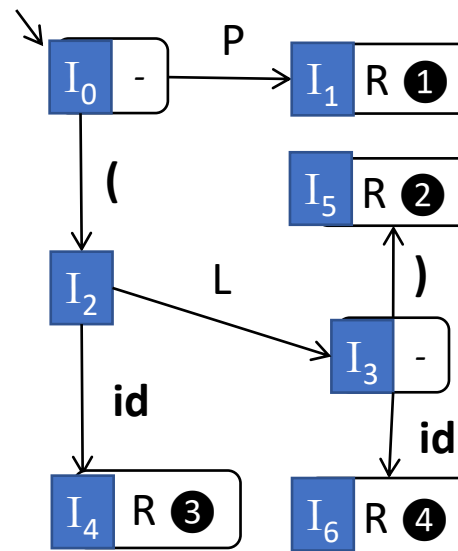
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Correct Parse Tree

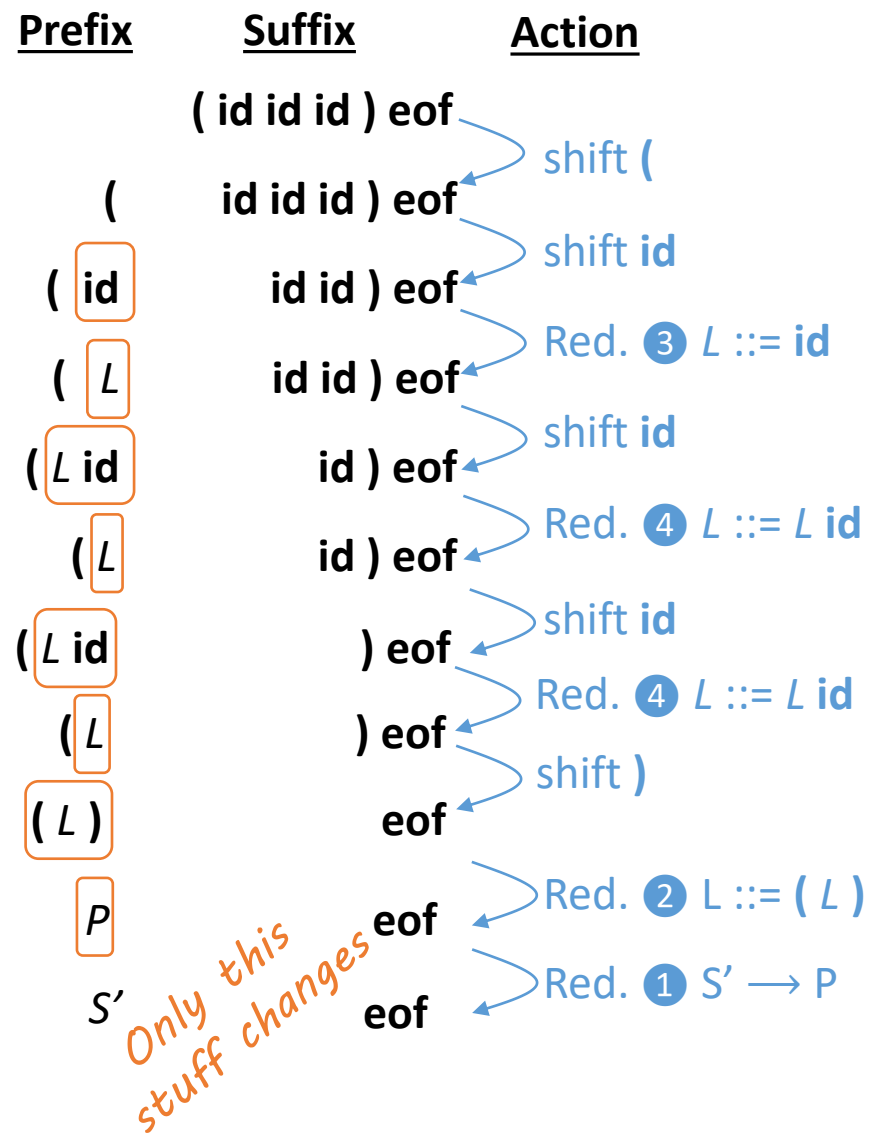


G Parser Automaton

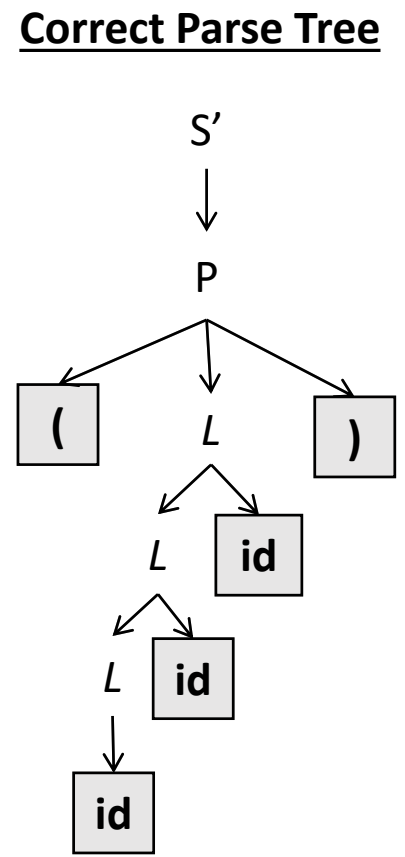
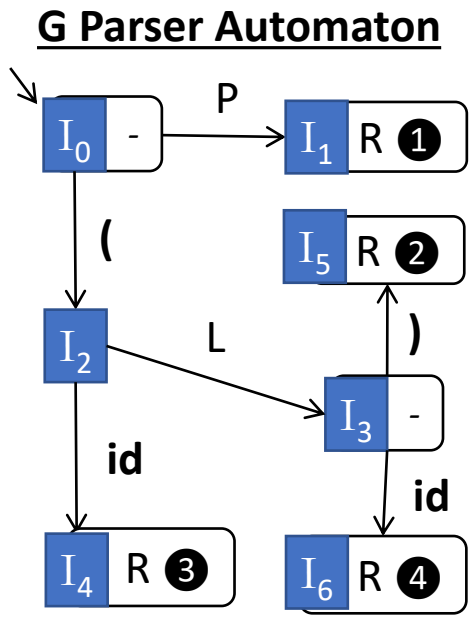


Shortcut: Don't Restart DFA Every Time

LR Parser Construction



- Grammar G**
- ① $S' ::= P$
 - ② $P ::= (L)$
 - ③ $L ::= id$
 - ④ $L ::= L id$



LR Parser Stack

LR Parser Construction

Amazing fact #2: Can keep position in the parse tree *and* position in the automaton in a single stack

- The prefix becomes implicit in the stack
- Can go back to tracking a lookahead “next” token just beyond the prefix

Advancing the automaton

- Terminal Edges – move terminal ~~symbol into prefix~~ Item onto stack
- Nonterminal Edges – add nonterminal ~~symbol to prefix~~
- Accepting states – ~~modify the existing prefix~~ Item onto stack
Pop RHS stack items

LR Parser Stack

LR Parser Construction

Input

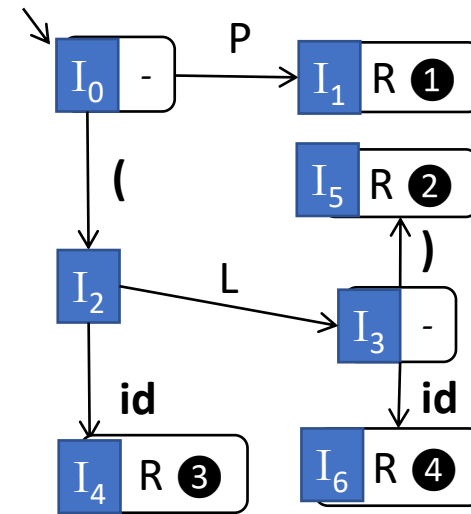
(id id) eof

$T_1 T_2 T_3 T_4 T_5$

Grammar G

- 1 $S' ::= P$
- 2 $P ::= (L)$
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- 4 $L ::= L id$

G Parser Automaton



Terminal edge

$I_0, (\rightarrow I_2$

Terminal edge

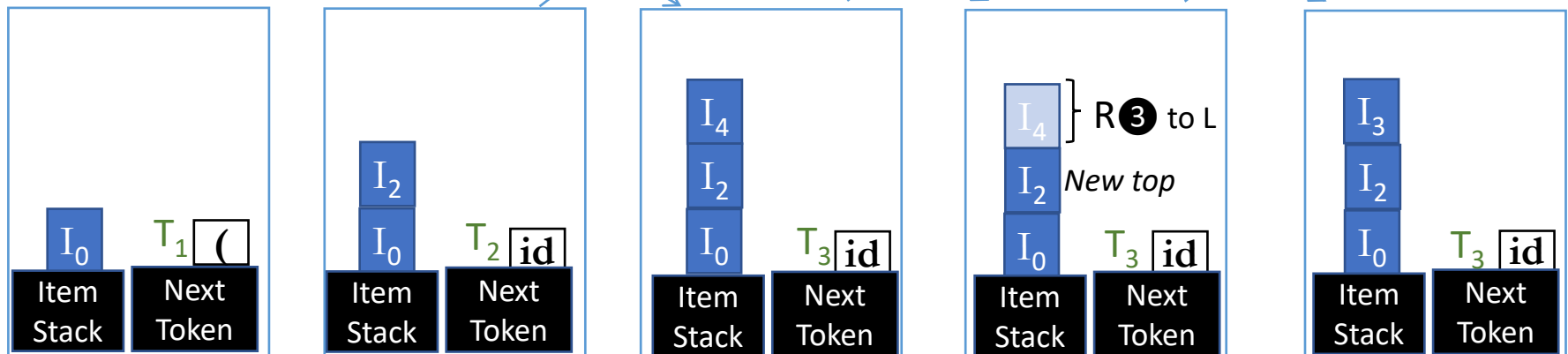
$I_2, id \rightarrow I_4$

Accepting state

Pop 1 stack item

Nonterminal edge

$I_2, L \rightarrow I_3$



LR Parser Stack

LR Parser Construction

Input

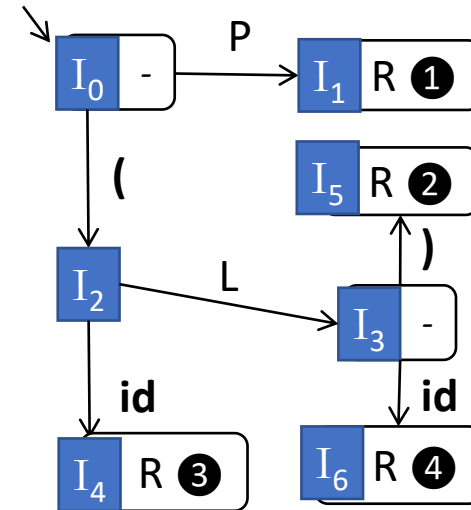
(id id) eof

$T_1 T_2 T_3 T_4 T_5$

Grammar G

- ① $S' ::= P$
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G Parser Automaton



Terminal edge

$I_3, id \rightarrow I_6$

Accepting state ④

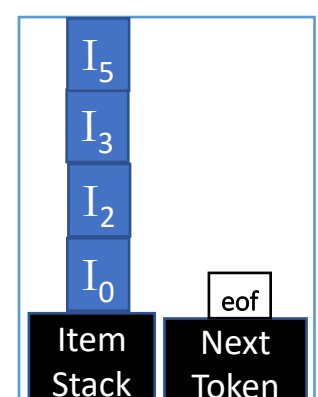
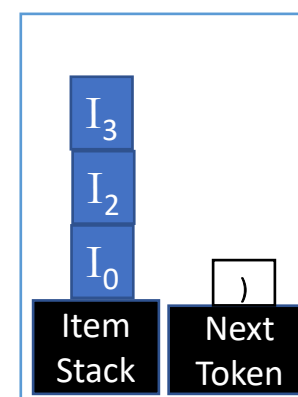
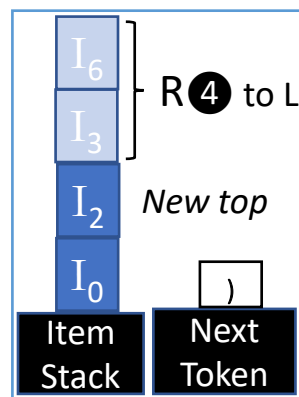
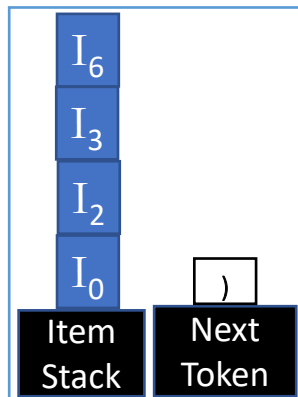
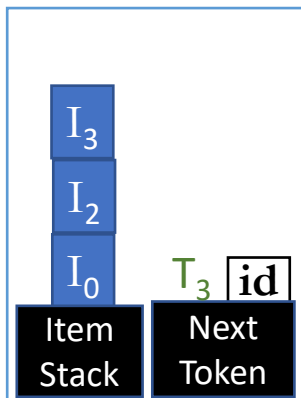
Pop 2 items

Terminal edge

$I_2, L \rightarrow I_3$

Terminal edge

$I_3,) \rightarrow I_5$



LR Parser Stack

LR Parser Construction

Input

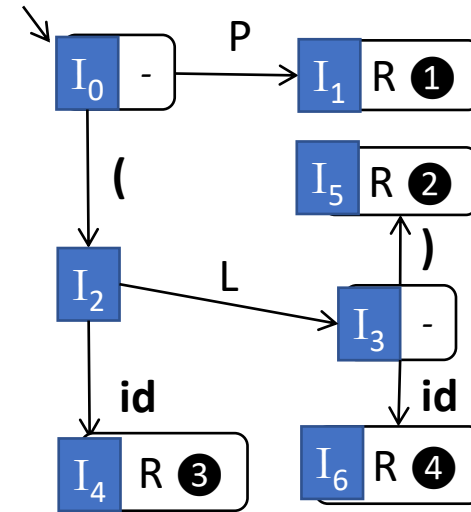
(id id) eof

$T_1 T_2 T_3 T_4 T_5$

Grammar G

- ① $S' ::= P$
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G Parser Automaton



Accept state ②

Nonterminal edge

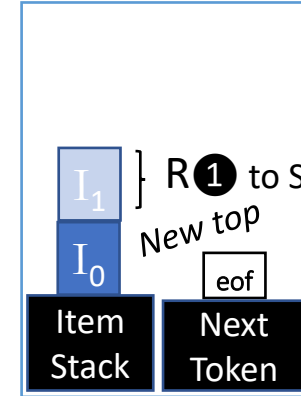
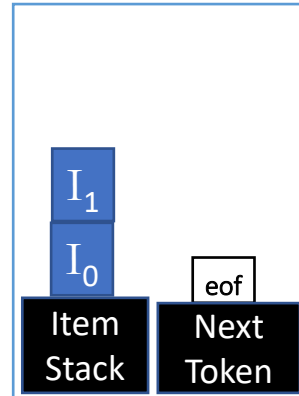
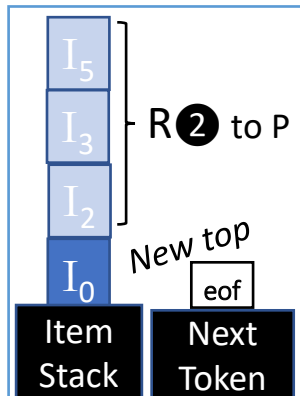
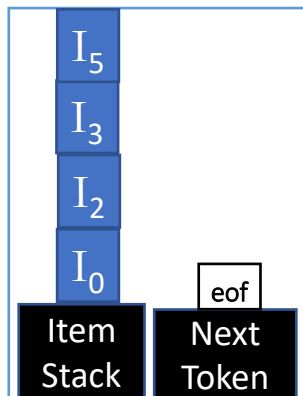
Accept state ①

Pop 3 items

$I_3,) \rightarrow I_5$

Pop 1 item

$I_0, S' \rightarrow \text{Accept!}$



Next: Building the Parser

LR Parser Construction

Final Tasks to complete Implementation:

- Building the “Prefix” Automaton
- Translating the Automaton into a table