

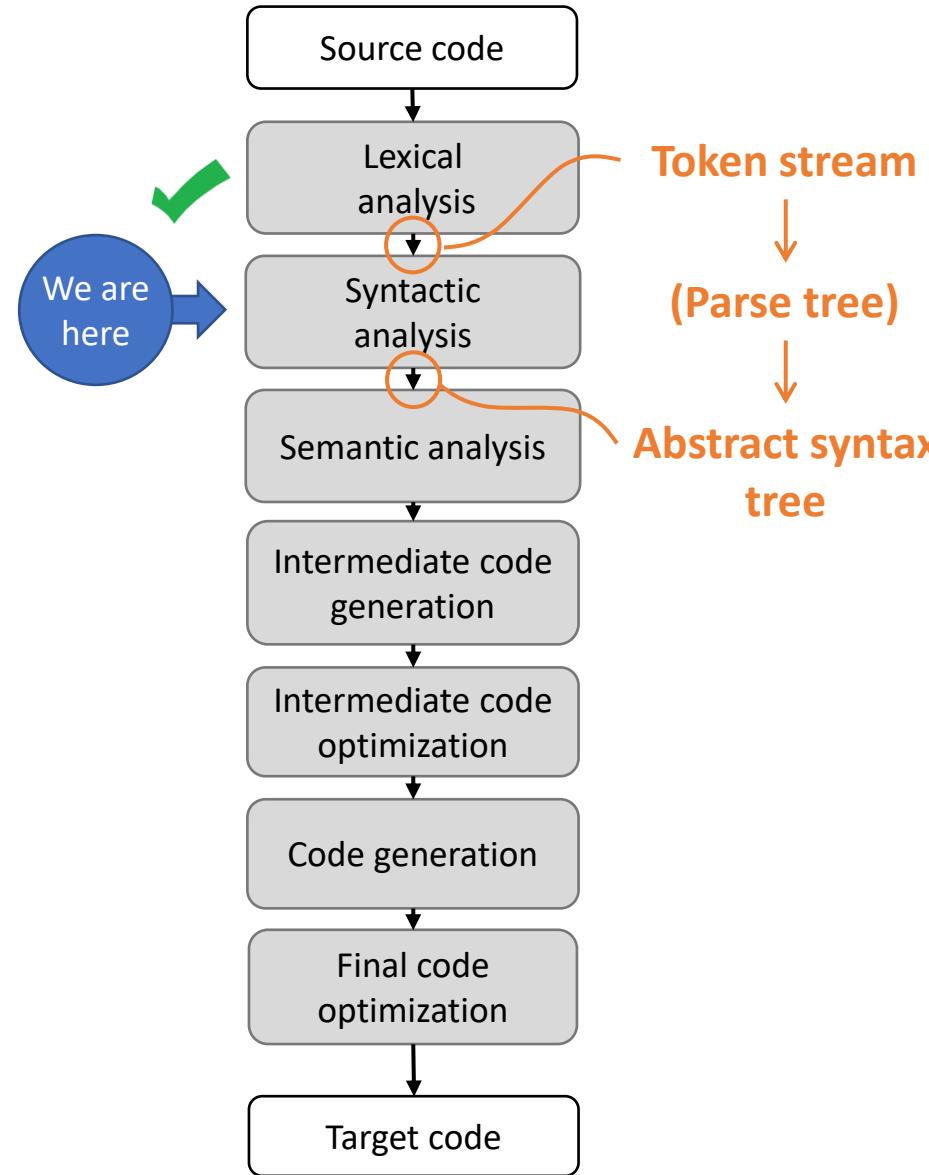
University of Kansas | Drew Davidson

ECS 665 COMPILER CONSTRUCTION

(Predictive) Parsing

Compiler Construction

Progress Pics



Syntactic Analysis

- We know **what** we want (induce an AST from a given token stream)

Last Time

Review - ASTs

Syntax-Directed Definition

- Showed how to use SDD to induce program's AST
- We can *specify* a correct syntactic analysis

You Should Know

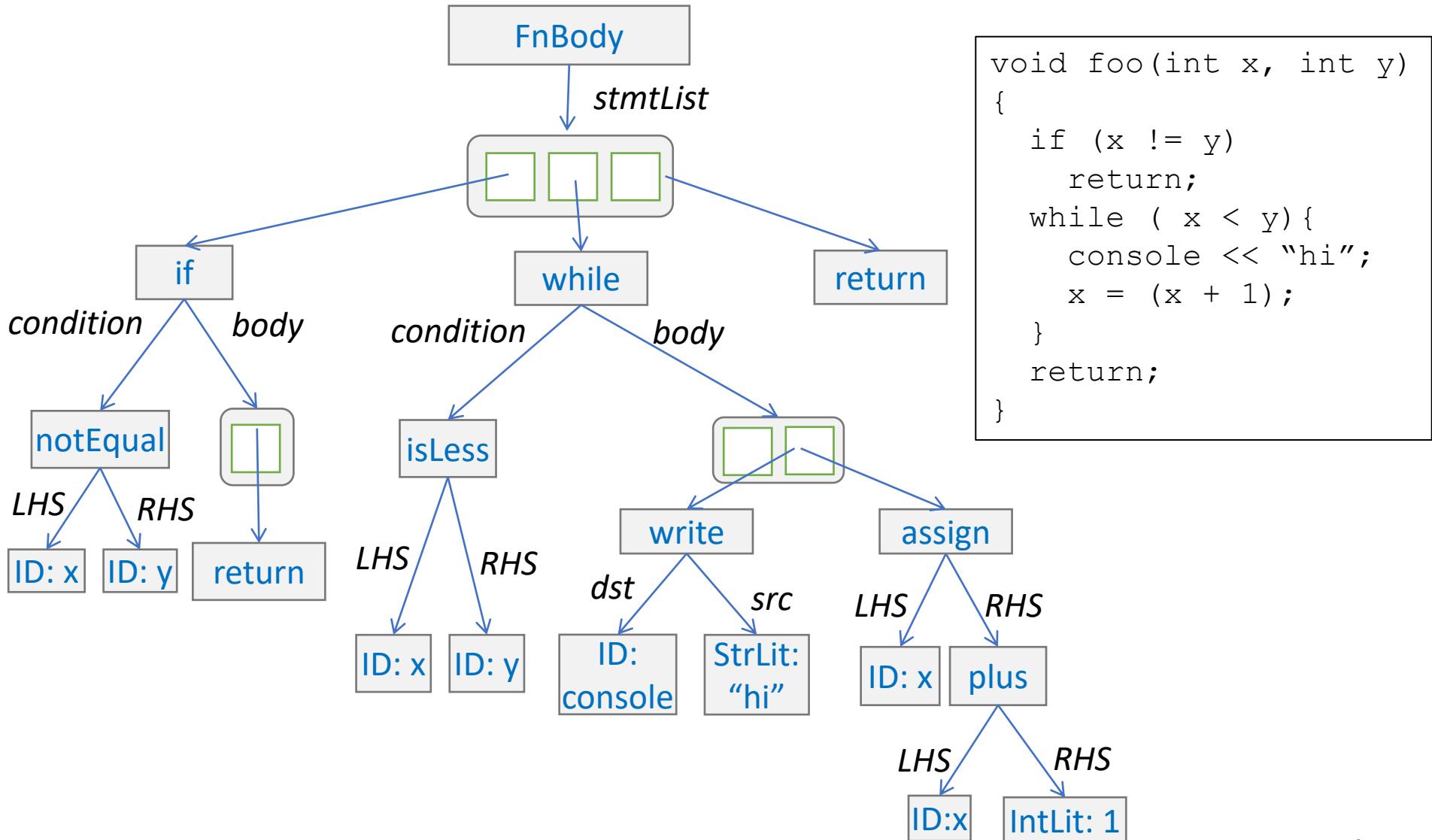
- The concepts of Parse Trees / ASTs
- How to specify syntax-directed definitions (to induce scalar values, data structures, etc. from parse trees)



**Syntax-Directed
Definition**

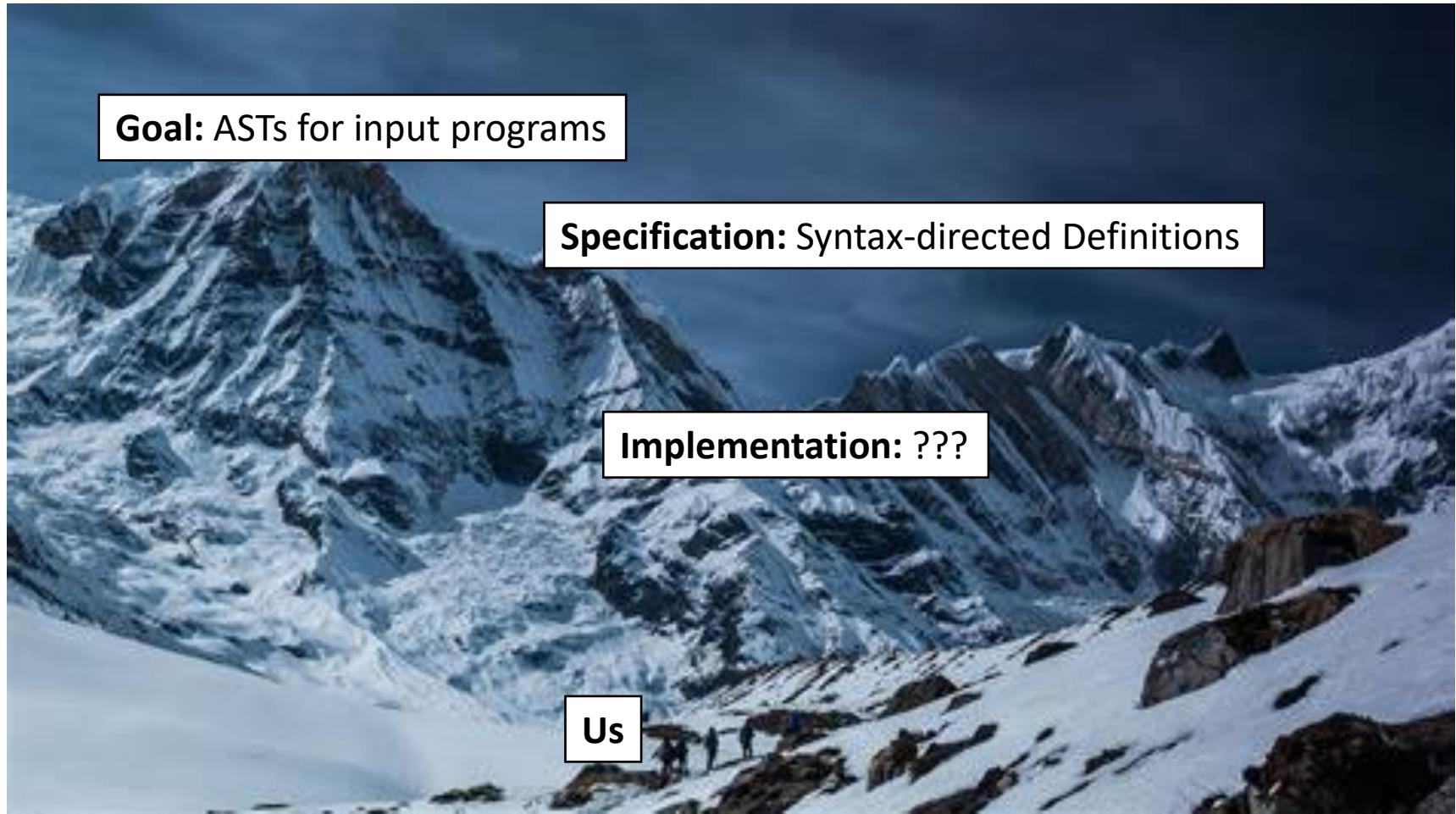
AST For Input Code Snippet

Abstract Syntax Trees – ASTs in Code



The Goal and The Path

Abstract Syntax Trees – ASTs in Code



COMPILER

LAND
+ HAMILTON GRAPEYLAND



Today's Outline

(Predictive) Parsing

Parsing

- Complexity

The LL(k) Languages

- Intro
- LL(1) parsing
- LL(1) transformations



Parsing

The Price of Expressive Power

(Predictive) Parsing – Parsing Complexity

Parsing is costly

- Jumping from Regular Languages to CF-Languages incurs penalties
 - Lose lots of language properties
 - Complexity for some operations goes up



Power always comes at a price

Parsing: Bad News/Good News

(Predictive) Parsing – Parsing Complexity

Bad News

- Complexity - CFL recognition:
 - CYK: $O(n^3)$
 - Best known bound: $O(n^{2.3728639})$
- Parsing is at least as hard



Good News

- We don't need to recognize all CFLs
 - We can assume ambiguity-free syntax
 - Exchange expressiveness for faster parsers

Scaling Back Expressive Power

(Predictive) Parsing – Parsing Complexity

Compilers aren't concerned with arbitrary languages

- Programming languages *should* be unambiguous
- Complex operations can be built from simple syntax

Paying for features we don't need!



“Don’t buy the truck if you ain’t haulin’ freight”
-Drew Davidson

Restricting the Language Class

(Predictive) Parsing – Parsing Complexity

- Allows us to...
 - Build linear-time parsers
 - Detect ambiguity
- Let's consider one such class of restrictions



Disallowing Heavyweight Languages

Today's Outline

Preview Lecture 8 – Predictive Parsing

Parsing

- Complexity

The LL(k) languages

- Intro
- LL(1) parsing
- LL(1) transformations

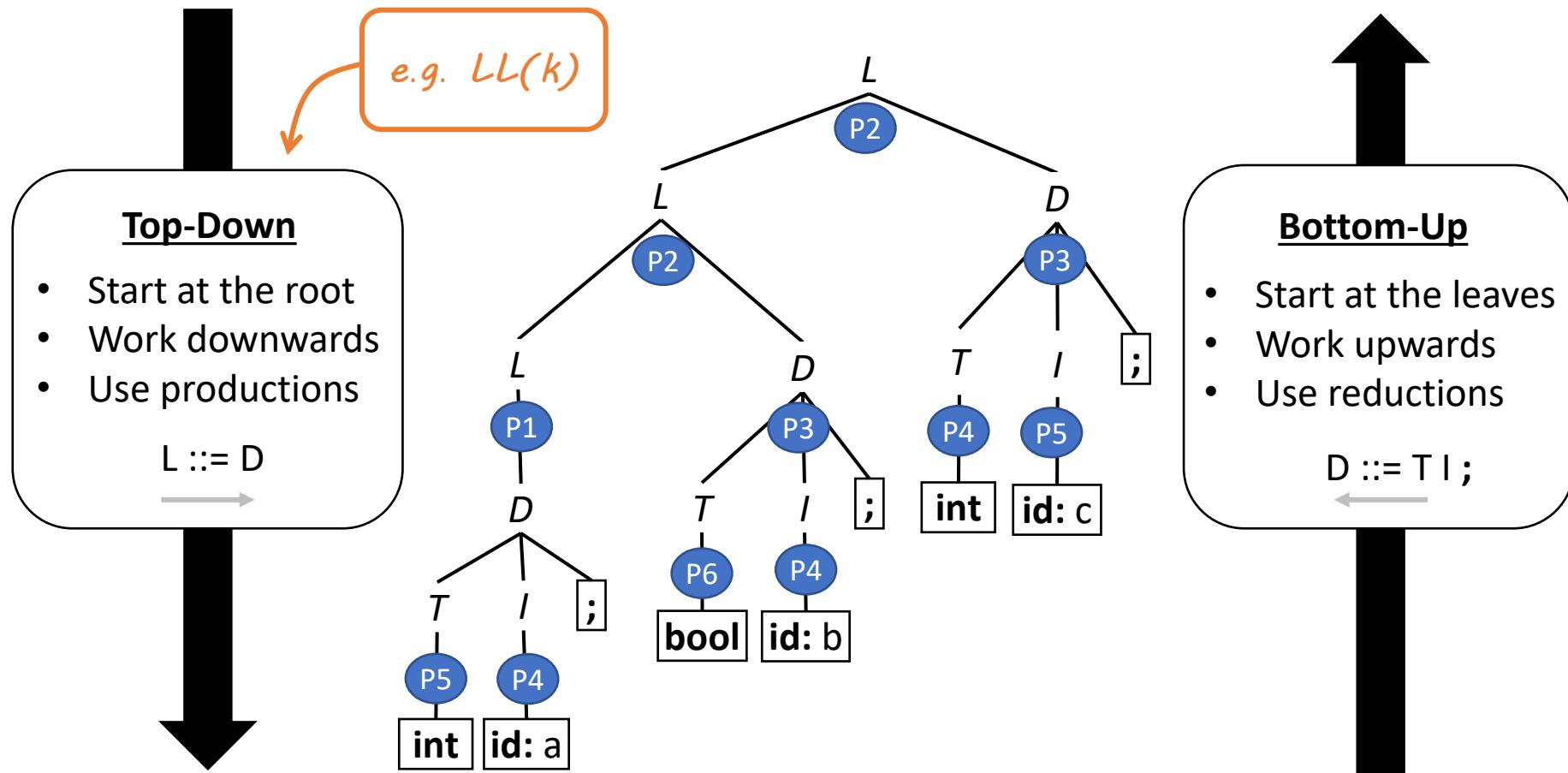


*Toyota Yaris: A sensible compact
LL(1) languages are the Yaris of the
Parsing world*

Parser Type

LL(k) Parsers - Definitions

Most parsers classified by how they build the parse tree



Predictive Parsers

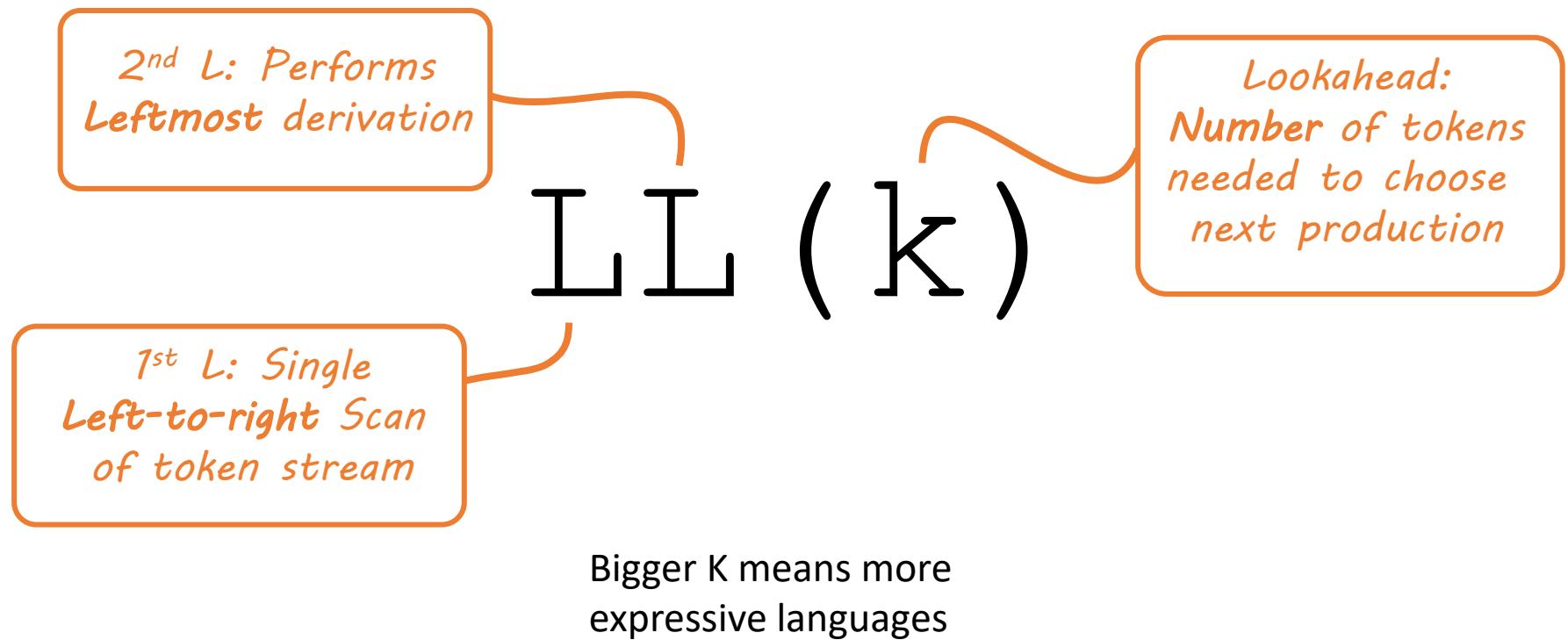
LL(k) Parsers - Definitions

- “Predict” correct production for the given nonterminal before seeing all RHS symbols
- Based on input prefix “Guess and check” the suffix



LL Grammars: Anatomy of Notation

LL Grammars - Intro



LL Grammar Examples

LL Grammars - Intro

LL(0) Grammars

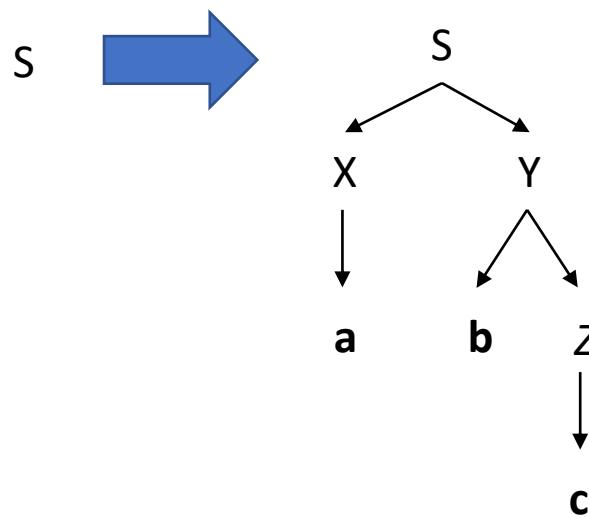
Takes leftmost derivation
step knowing **zero**
underived input tokens

$S ::= X Y$

$X ::= a$

$Y ::= b Z$

$Z ::= c$



LL Grammar Examples

LL Grammars - Intro

LL(0) Grammars

Takes leftmost derivation
step knowing **zero**
underived input tokens

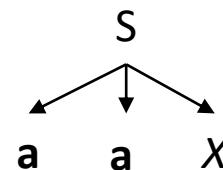
$$S ::= X Y$$
$$X ::= a$$
$$Y ::= b Z$$
$$Z ::= c$$

LL(1) Grammars

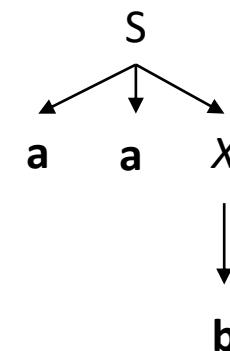
Takes leftmost derivation
step knowing **one**
underived input token

$$S ::= a a X$$
$$\mid b X$$
$$X ::= a$$
$$\mid b$$

S
Lookahead is a



Lookahead is b

LL Grammar Examples

LL Grammars - Intro

LL(0) Grammars

Takes leftmost derivation
step knowing **zero**
underived input tokens

$$\begin{aligned} S &::= X Y \\ X &::= a \\ Y &::= b Z \\ Z &::= c \end{aligned}$$

LL(1) Grammars

Takes leftmost derivation
step knowing **one**
underived input token

$$\begin{aligned} S &::= a a X \\ &\quad | \quad b X \\ X &::= a \\ &\quad | \quad b \end{aligned}$$

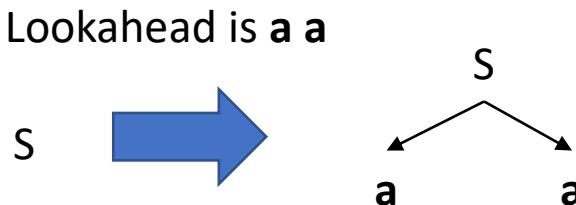
LL(2) Grammars

Takes leftmost derivation
step knowing **two**
underived input tokens

$$\begin{aligned} S &::= a a \\ &\quad | \quad a b \end{aligned}$$

(Lookahead **a** is insufficient)

Lookahead is **a a**



LL Grammar Examples

LL Grammars - Intro



LL(1) Grammars

Takes leftmost derivation
step knowing **one**
underived input token

Row:
leftmost leaf
nonterminal

	Column Lookahead token			
	Cell: Production			

LL Grammar Examples

LL Grammars - Intro

With some grammars you can
just eyeball the selector table

$$\begin{aligned} S ::= & (S) \\ & | \{ S \} \\ & | \epsilon \end{aligned}$$

		Column Lookahead token			
		({)	}
Row: leftmost leaf nonterminal	S	(S)	{ S }	ϵ	ϵ

Cell:
Production

Today's Outline

(Predictive) Parsing

Parsing

- Complexity

A New Type of Language - LL

- Intro
- LL(1) parsing
- LL(1) transformations



Parsing

LL(1) Algorithm Psuedocode

Predictive Parsing - LL(1) Parsing

```
stack.push(eof)
stack.push(Start non-term)
lookahead = scanner.first_token()
Repeat
    if stack.top is a terminal
        match stack.top with lookahead
        pop y from the stack
        lookahead = scanner.next_token()
    if stack.top is a nonterminal
        X = stack.pop()
        get P = table[X,lookahead]
        push P's RHS symbols Right-to-Left
Until one of the following:
    stack is empty (accept)
    stack.top is a terminal that doesn't match t (reject)
    stack.top is a non-term and table entry is empty (reject)
```

}

Initialization

*Case 1
Terminal on stack:
check prediction*

*Case 2
Non-terminal on stack:
Derive new prediction*

Exit

Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Initialization

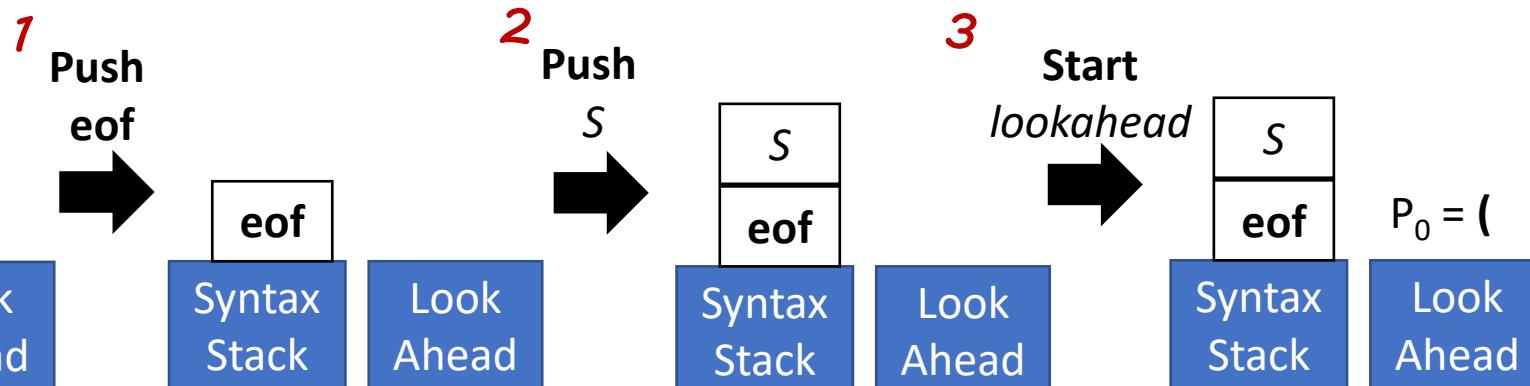
- 1 stack.push(**eof**)
- 2 stack.push(*Start* non-term)
- 3 lookahead = scanner.first_token()

Selector Table

	()	{	}
S	(S)	ϵ	$\{S\}$	ϵ

Input Stream

({	})	.	eof
P_0	P_1	P_2	P_3	P_4	



Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Initialization

- 1 stack.push(**eof**)
- 2 stack.push(*Start non-term*)
- 3 lookahead = scanner.first_token() "Predicted"

Selector Table

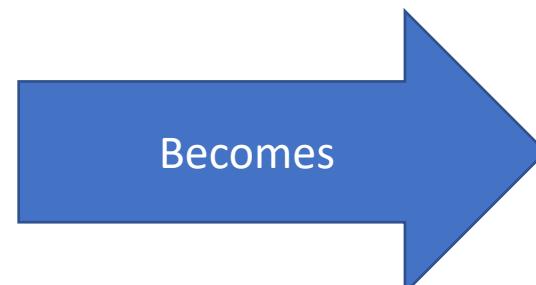
	()	{	}
S	(<i>S</i>)	ϵ	{ <i>S</i> }	ϵ

Input Stream

({	})	eof
P ₀	P ₁	P ₂	P ₃	P ₄

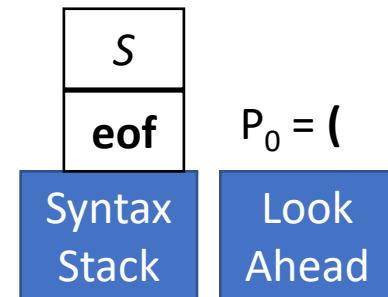
Derivation Sequence

S



Syntax Stack

Look Ahead



Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Nonterminal on stack:

- 1 X = pop(stack)
- 2 get P = table[X, lookahead]
- 3 push P's symbols Right-to-Left

"Predicted"

Selector Table

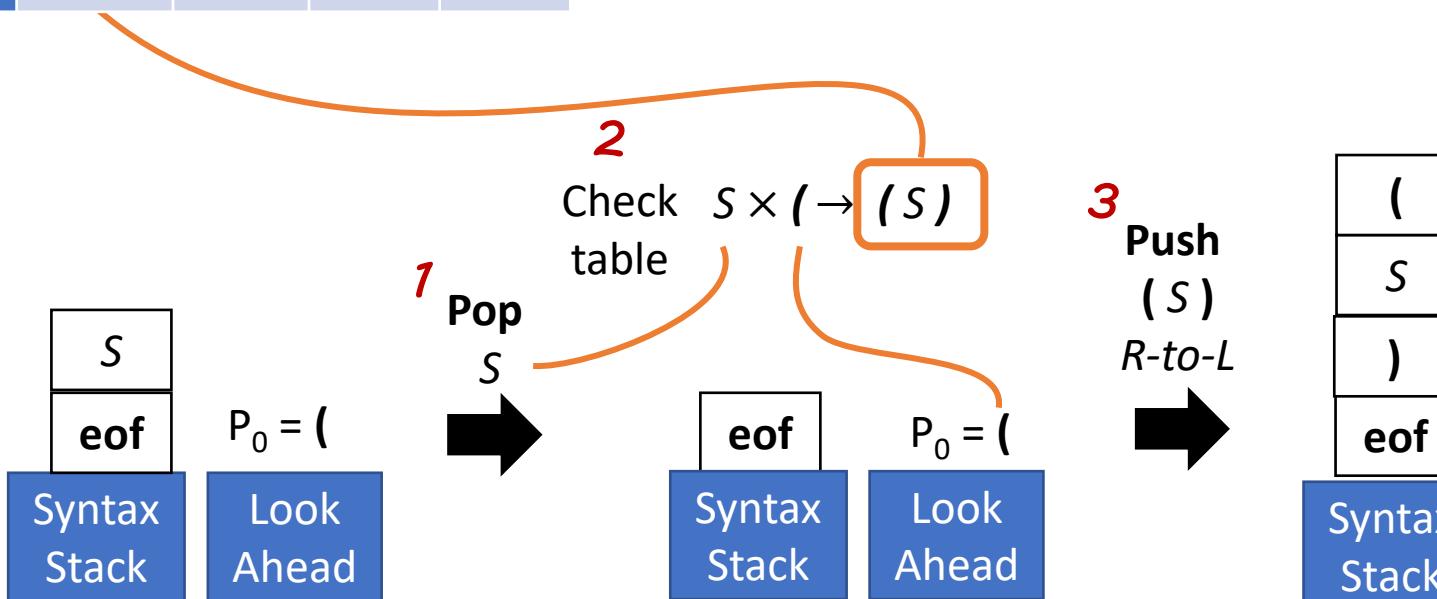
	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({ }) eof
P₀ P₁ P₂ P₃ P₄

Derivation Sequence

S



Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Nonterminal on stack:

1 X = pop(stack)

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3 push P's symbols Right-to-Left

"Predicted"

Selector Table

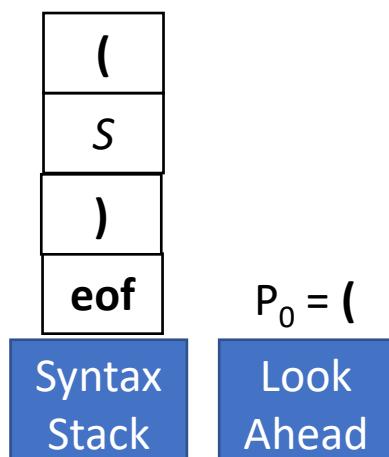
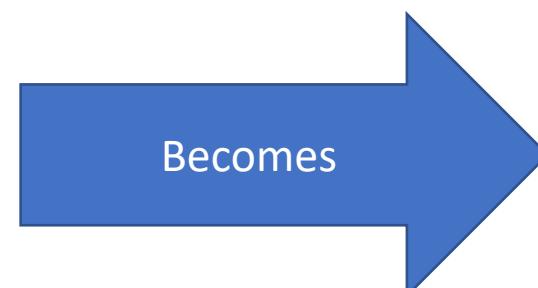
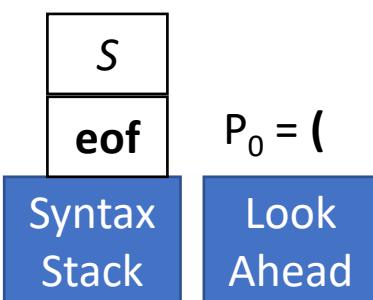
	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({ }) eof
 $P_0 \quad P_1 \quad P_2 \quad P_3 \quad P_4$

Derivation Sequence

$S \Rightarrow (S)$



Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

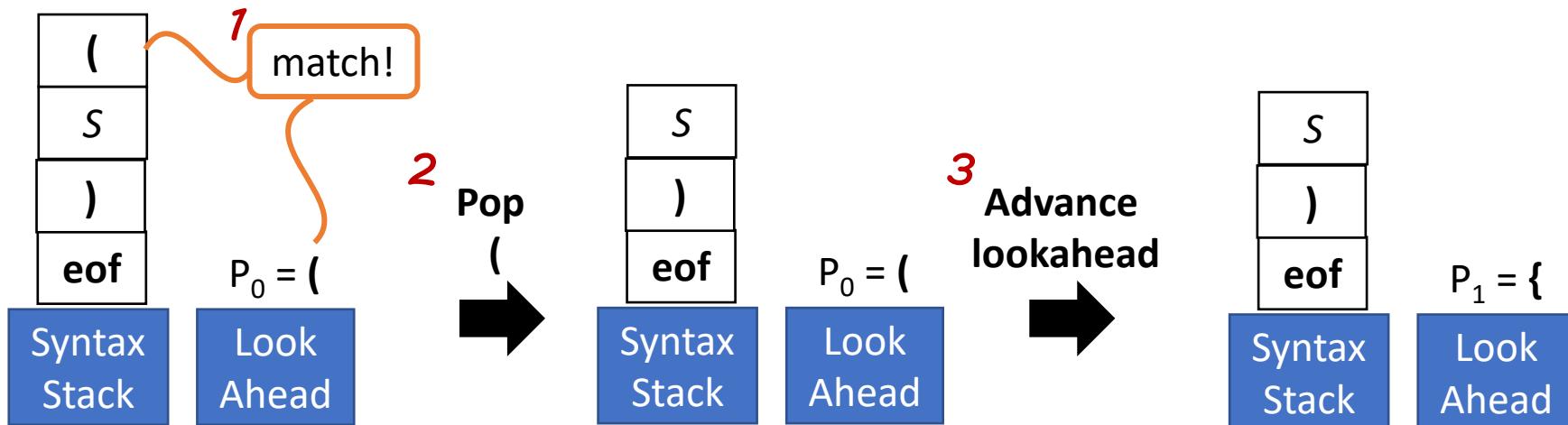
- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next_token() "Predicted"

Input Stream

({ }) eof
 $P_0 \quad P_1 \quad P_2 \quad P_3 \quad P_4$

Derivation Sequence

$S \Rightarrow (S)$



Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next_token() "Predicted"

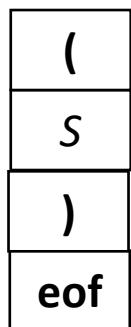
Input Stream

({ }) eof
 P_0 P_1 P_2 P_3 P_4



Derivation Sequence

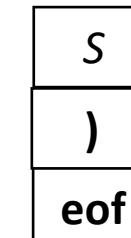
$S \Rightarrow (S)$



$P_0 = ($

Syntax Stack

Look Ahead



Syntax Stack

$P_1 = \{$

Look Ahead

Example Grammar

$$S ::= (S) \\ | \{ S \} \\ | \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing
Nonterminal on stack:

- 1 $X = \text{pop}(\text{stack})$
- 2 get $P = \text{table}[X, \text{lookahead}]$
- 3 push P' 's RHS symbols *Right-to-Left*

"Predicted"

Selector Table

	()	{	}
S	(S)	ϵ	$\{S\}$	ϵ

Input Stream

({ }) eof
 $P_0 \quad P_1 \quad P_2 \quad P_3 \quad P_4$
✓

Derivation Sequence

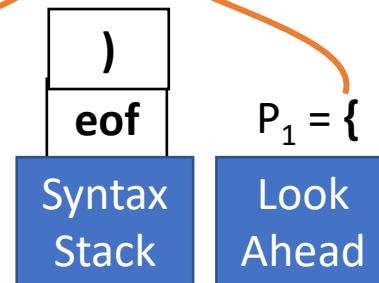
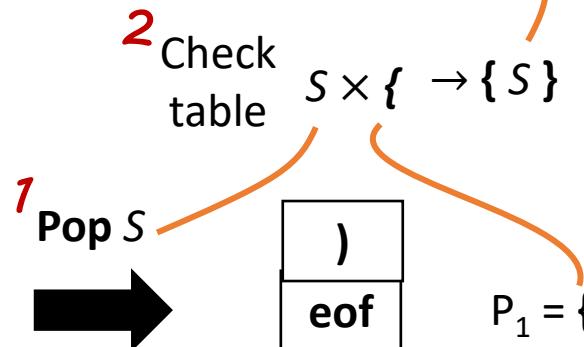
$S \Rightarrow (S)$

S
)
eof

$P_1 = \{$

Syntax Stack

Look Ahead



{
S
}
)

Syntax Stack
Look Ahead

$P_1 = \{$
Look Ahead

Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Nonterminal on stack:

- 1 X = pop(stack)
- 2 get P = table[X, lookahead]
- 3 push P's RHS symbols *Right-to-Left*

"Predicted"

Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

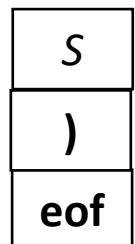
Input Stream

({ }) eof
 P_0 P_1 P_2 P_3 P_4



Derivation Sequence

$S \Rightarrow (S)$
 $\Rightarrow (\{ S \})$

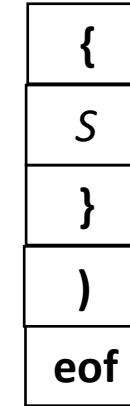


$P_1 = \{$

Syntax Stack

Look Ahead

Becomes



$P_1 = \{$

Syntax Stack

Look Ahead

Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next_token()

"Predicted"

Selector Table

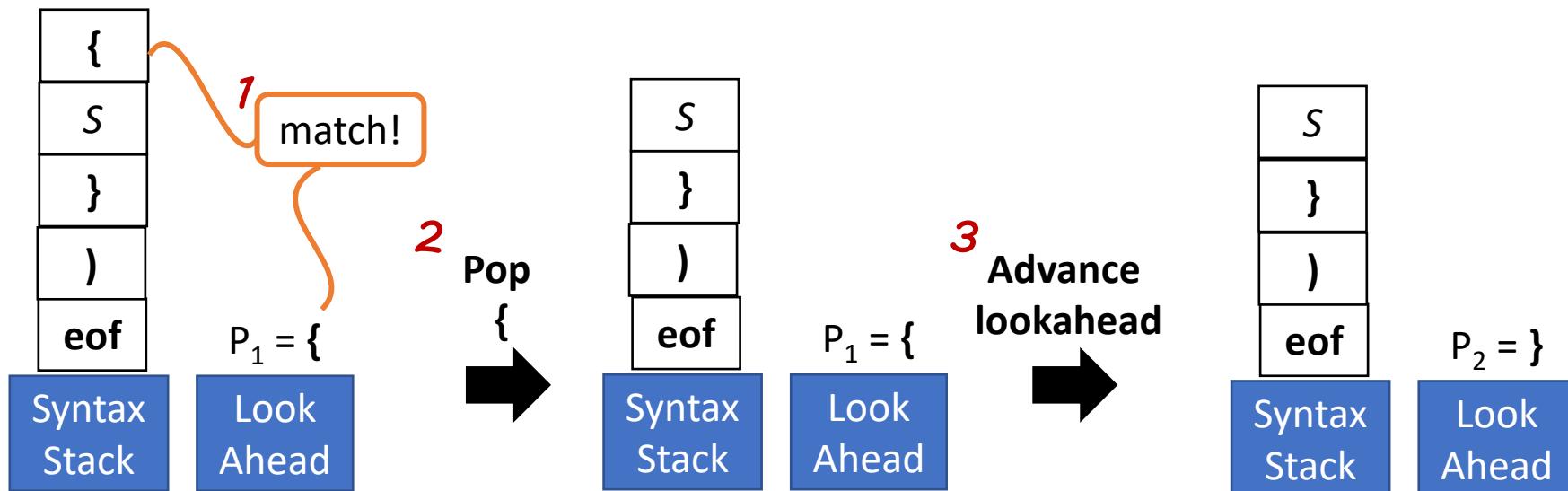
	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({ }) eof
 $P_0 \quad P_1 \quad P_2 \quad P_3 \quad P_4$

Derivation Sequence

$S \Rightarrow (S)$
 $\Rightarrow (\{ S \})$



Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next_token()

"Predicted"

Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({	})	eof
P_0	P_1	P_2	P_3	P_4

Derivation Sequence

$$S \Rightarrow (S)$$

$$\Rightarrow (\{ S \})$$

{
S
}
)
eof

$P_1 = \{$

Syntax Stack

Look Ahead

Becomes

S
}
)
eof

$P_2 = \}$

Syntax Stack

Look Ahead

Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Nonterminal on stack:

- 1 X = pop(stack)
- 2 get P = table[X, lookahead]
- 3 push P's RHS symbols Right-to-Left "Predicted"

Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({	})	eof
P ₀	P ₁	P ₂	P ₃	P ₄

Derivation Sequence

$$S \Rightarrow (S)$$

$$\Rightarrow (\{S\})$$

2 Check table
 $S \times \{ \rightarrow \epsilon$

1 Pop S

3 Push ϵ (nothing)

S
}
)
eof

Syntax Stack

Look Ahead

}
)
eof

Syntax Stack
Look Ahead

}
)
eof

Syntax Stack
Look Ahead

P₂ = }

Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Nonterminal on stack:

1 X = pop(stack)

2 get P = table[X, lookahead]

3 push P's RHS symbols *Right-to-Left* "Predicted"

Selector Table

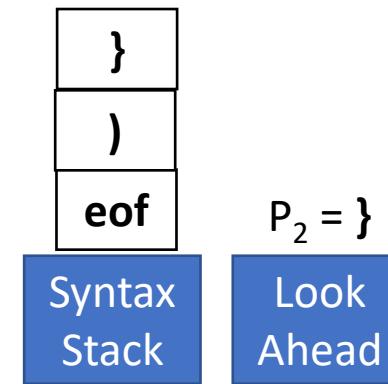
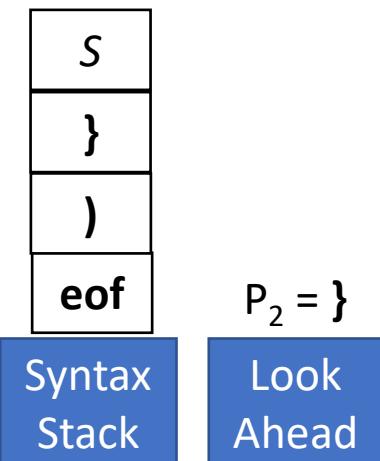
	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({ }) eof
 P_0 P_1 P_2 P_3 P_4
✓ ✓

Derivation Sequence

$S \Rightarrow (S)$
 $\Rightarrow (\{ S \})$
 $\Rightarrow (\{ \})$



Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next_token()

"Predicted"

Selector Table

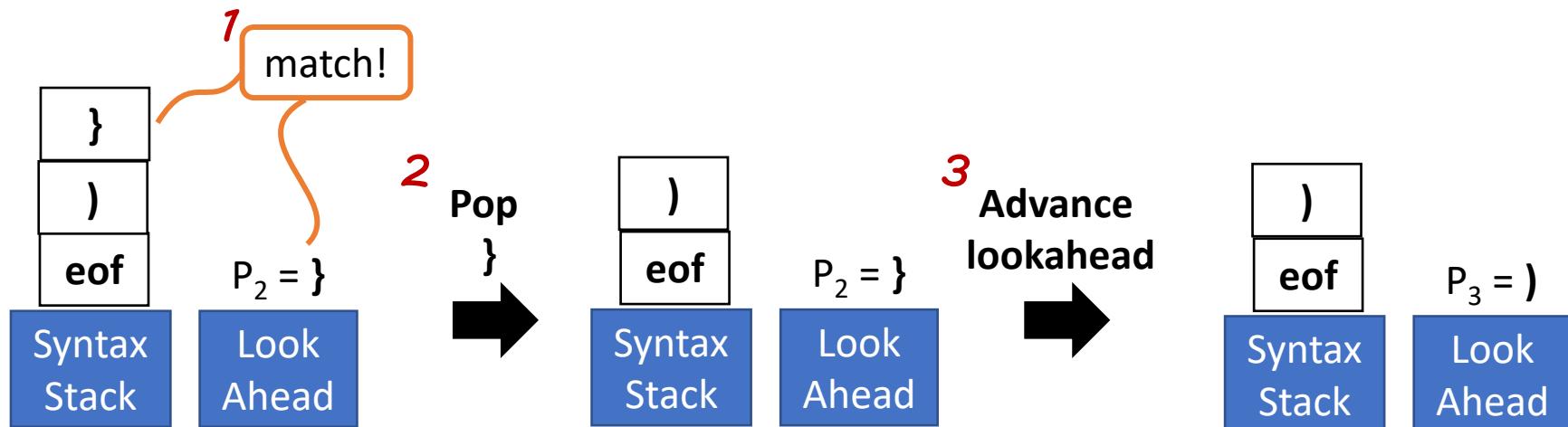
	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({ }) eof
 P_0 P_1 P_2 P_3 P_4
✓ ✓ ✓

Derivation Sequence

$S \Rightarrow (S)$
 $\Rightarrow (\{ S \})$
 $\Rightarrow (\{ \})$



Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next_token()

“Predicted”

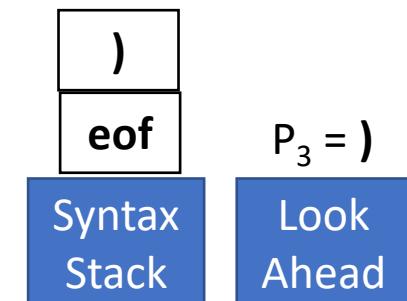
Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({	})	eof
P_0	P_1	P_2	P_3	P_4
✓	✓	✓		

Derivation Sequence

$$\begin{aligned} S &\Rightarrow (S) \\ &\Rightarrow (\{ S \}) \\ &\Rightarrow (\{ \}) \end{aligned}$$


Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next_token()

"Predicted"

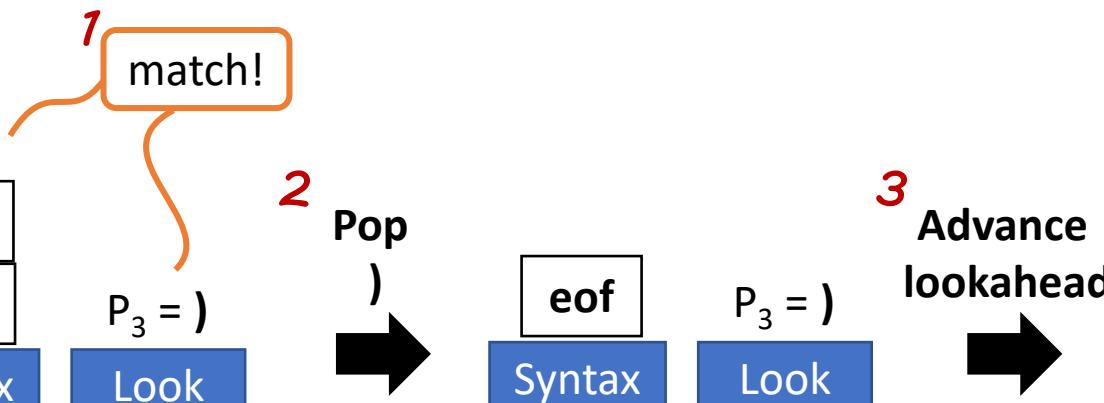
Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({	})	eof
P ₀	P ₁	P ₂	P ₃	P ₄
✓	✓	✓		

Derivation Sequence

$$\begin{aligned} S &\Rightarrow (S) \\ &\Rightarrow (\{ S \}) \\ &\Rightarrow (\{ \}) \end{aligned}$$


Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next_token()

“Predicted”

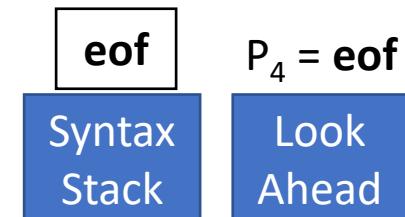
Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({	})	eof
P_0	P_1	P_2	P_3	P_4
✓	✓	✓	✓	

Derivation Sequence

$$\begin{aligned} S &\Rightarrow (S) \\ &\Rightarrow (\{ S \}) \\ &\Rightarrow (\{ \}) \end{aligned}$$


Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next_token()

"Predicted"

Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

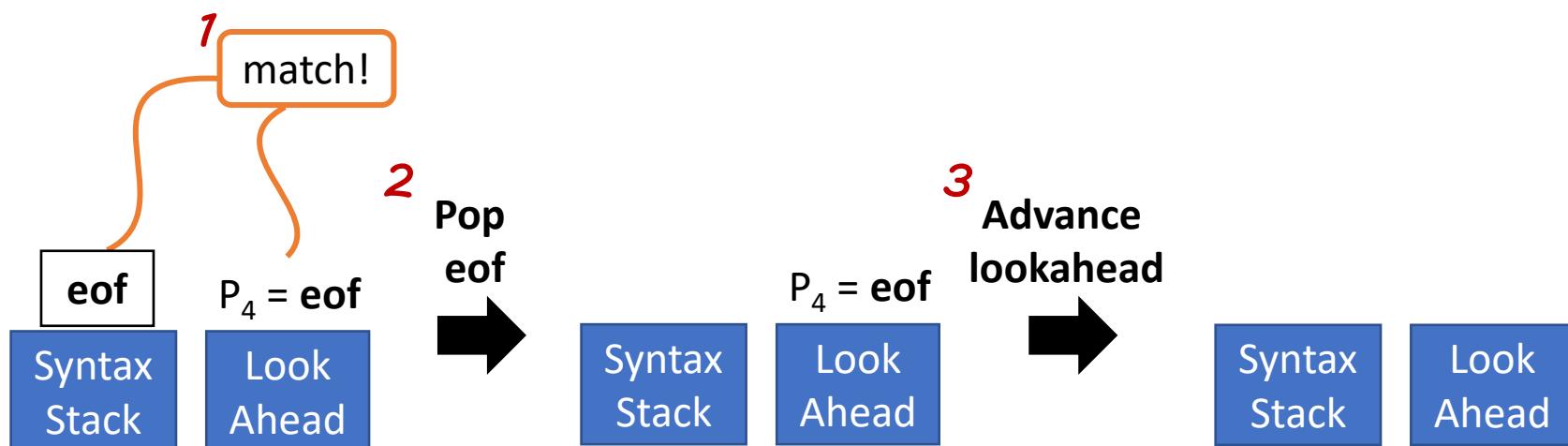
Input Stream

({	})	eof
P_0	P_1	P_2	P_3	P_4
✓	✓	✓	✓	✓

Derivation Sequence

$$S \Rightarrow (S)$$

$$\Rightarrow (\{ S \})$$

$$\Rightarrow (\{ \})$$


Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Terminal on stack:

- 1 match stack.top with lookahead
- 2 pop stack
- 3 lookahead = scanner.next_token()

“Predicted”

Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

({	})	eof
P_0	P_1	P_2	P_3	P_4
✓	✓	✓	✓	✓

Derivation Sequence

$$S \Rightarrow (S)$$

$$\Rightarrow (\{ S \})$$

$$\Rightarrow (\{ \})$$

Syntax
Stack

Look
Ahead

Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Example

(Predictive) Parsing - LL(1) Parsing

Exit

Stack and Lookahead both empty:
Accept!

“Predicted”

Selector Table

	()	{	}
S	(S)	ϵ	$\{S\}$	ϵ

Input Stream

({	})	eof
P_0	P_1	P_2	P_3	P_4
$\Rightarrow \{\}$				

Derivation Sequence

$$S \Rightarrow (S)$$

$$\Rightarrow (\{S\})$$

$$\Rightarrow (\{\})$$


Syntax
Stack

Look
Ahead

Running LL(1)

(Predictive) Parsing - LL(1) Parsing

Linear parse!

- Once we verify a position we never look at it again

Pretty fast given general parsing complexity

- Especially once you start combining steps

Let's do another quick example

- This time we'll show rejecting bad input

Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Reject Example

(Predictive) Parsing - LL(1) Parsing

Selector Table

	()	{	}
S	(S)	ϵ	{ S }	ϵ

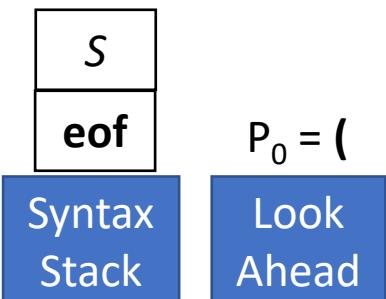
Input Stream

(} eof
 P_0 P_1 P_2

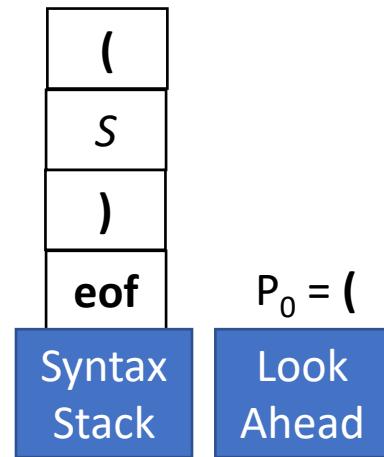
“Predicted”
Derivation Sequence

 $S \Rightarrow (S)$
Initialization

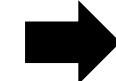
1,2,3



Nonterminal Actions
1,2,3



Terminal Actions
1,2,3



Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Reject Example

(Predictive) Parsing - LL(1) Parsing

Selector Table

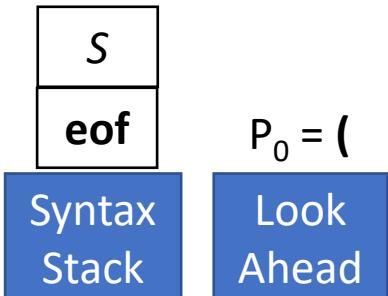
	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

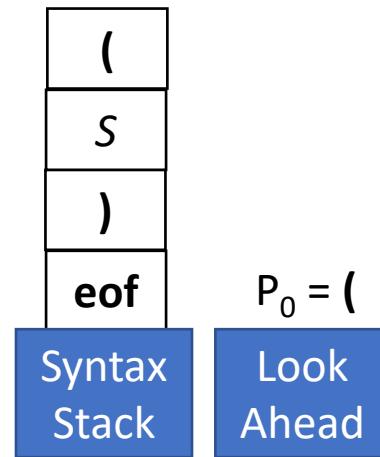
(} eof
 P_0 P_1 P_2

“Predicted”
Derivation Sequence

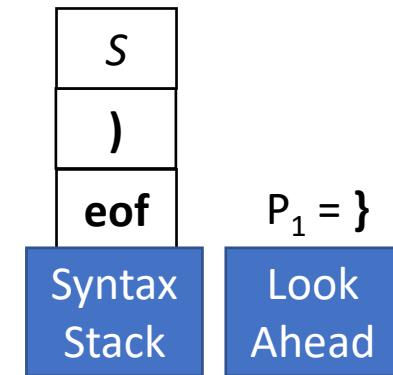
$S \Rightarrow (S)$

*Initialization**1,2,3*

Nonterminal Actions
1,2,3



Terminal Actions
1,2,3



Example Grammar

$$S ::= (S)$$

$$\quad | \quad \{ S \}$$

$$\quad | \quad \epsilon$$

LL(1) Reject Example

(Predictive) Parsing - LL(1) Parsing

Selector Table

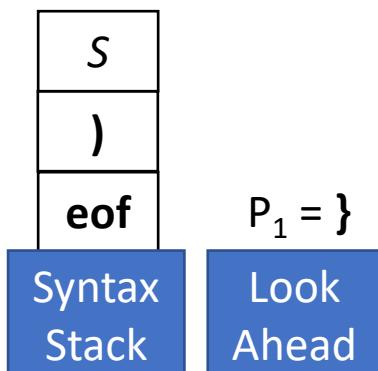
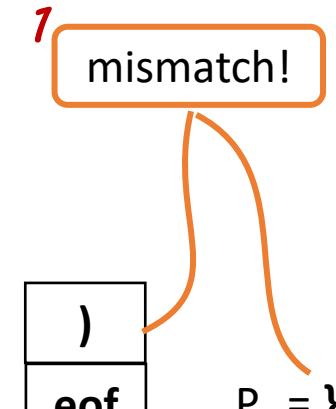
	()	{	}
S	(S)	ϵ	{ S }	ϵ

Input Stream

(} eof
 P_0 P_1 P_2

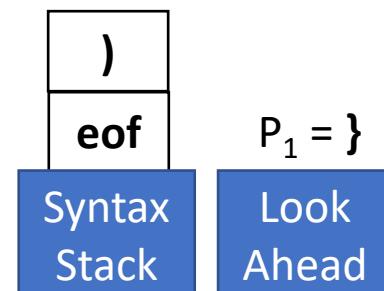
“Predicted” Derivation Sequence

$S \Rightarrow (S)$



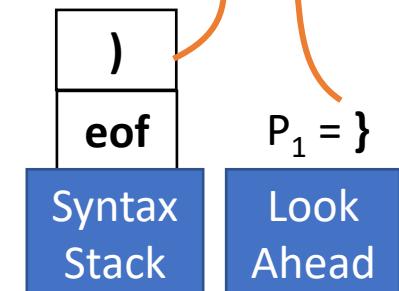
Nonterminal Actions

1,2,3



Terminal Actions

1 - Fails!



This Grammar is Great!

(Predictive) Parsing - LL(1) Parsing

With 1 token we know exactly what production it starts



...as long as the grammar is LL(1)

Consider a Concerning Grammar...

(Predictive) Parsing - LL(1) Parsing

Our prior
LL(2) Grammar

$$S ::= a \ a
| \ a \ b$$


Equivalent
LL(1) Grammar

$$S ::= a \ X
X ::= a | b$$

The Language
may be LL(1)
Even when the
Grammar is not LL(1)